## ΟΝΕ

# Introduction

#### ANSWERS TO REVIEW QUESTIONS

1. Guided missiles, automatic gain control in radio receivers, satellite tracking antenna

2. Yes - power gain, remote control, parameter conversion; No - Expense, complexity

3. Motor, low pass filter, inertia supported between two bearings

**4.** Closed-loop systems compensate for disturbances by measuring the response, comparing it to the input response (the desired output), and then correcting the output response.

5. Under the condition that the feedback element is other than unity

**6.** Actuating signal

**7.** Multiple subsystems can time share the controller. Any adjustments to the controller can be implemented with simply software changes.

8. Stability, transient response, and steady-state error

9. Steady-state, transient

**10.** It follows a growing transient response until the steady-state response is no longer visible. The system will either destroy itself, reach an equilibrium state because of saturation in driving amplifiers, or hit limit stops.

11. Natural response

12. Determine the transient response performance of the system.

**13.** Determine system parameters to meet the transient response specifications for the system.

**14.** True

15. Transfer function, state-space, differential equations

16. Transfer function - the Laplace transform of the differential equation

<u>State-space</u> - representation of an nth order differential equation as n simultaneous first-order differential equations

Differential equation - Modeling a system with its differential equation

### **SOLUTIONS TO PROBLEMS**









If the narrow light beam is modulated sinusoidally the pupil's diameter will also vary sinusoidally (with a delay see part c) in problem)

**c.** If the pupil responded with no time delay the pupil would contract only to the point where a small amount of light goes in. Then the pupil would stop contracting and would remain with a fixed diameter.



12. Vision Student Instructor System Position Position Tactile Student System 13. Steering wheel angle Steering Car Desired Roll Angle Controller Servo Dynamics Potentiometer Actual Roll Angle







**a.** 
$$L\frac{di}{dt} + Ri = u(t)$$

**b.** Assume a steady-state solution  $i_{ss} = B$ . Substituting this into the differential equation yields RB =

from which  $B = \frac{1}{R}$ . The characteristic equation is LM + R = 0, from which  $M = -\frac{R}{L}$ . Thus, the total solution is  $i(t) = Ae^{-(R/L)t} + \frac{1}{R}$ . Solving for the arbitrary constants,  $i(0) = A + \frac{1}{R} = 0$ . Thus,  $A = -\frac{1}{R}$ . The final solution is  $i(t) = \frac{1}{R} - \frac{1}{R}e^{-(R/L)t} = \frac{1}{R}(1 - e^{-(R/L)t})$ .



19.

**a.** Writing the loop equation,  $Ri + L\frac{di}{dt} + \frac{1}{C}\int idt + v_c(0) = v(t)$ **b.** Differentiating and substituting values,  $\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 16i = 0$ 

Writing the characteristic equation and factoring,

$$M^{2} + 2M + 16 = (M + 1 + \sqrt{15}i)(M + 1 - \sqrt{15}i)$$

The general form of the solution and its derivative is

$$i = A e^{-t} \cos(\sqrt{15t}) + B e^{-t} \sin(\sqrt{15t})$$
$$\frac{di}{dt} = (-A + \sqrt{15B}) e^{-t} \cos(\sqrt{15t}) - (\sqrt{15A} + B) e^{-t} \sin(\sqrt{15t})$$

18.

1,

Using 
$$i(0) = 0$$
;  $\frac{di}{dt}(0) = \frac{v_L(0)}{L} = \frac{1}{L} = 2$ 

$$i(0) = A = 0$$
 and  $\frac{di}{dt}(0) = -A + \sqrt{15}B = 2 \Longrightarrow B = \frac{2}{\sqrt{15}}$ 

c.

$$i(t) = \frac{2}{15}\sqrt{15} \ e^{-t}\sin(\sqrt{15} \ t)$$



20.

a. Assume a particular solution of

$$\mathbf{x}_{\mathrm{p}}(t) = C\cos(2t) + D\sin(2t)$$

Substitute into the differential equation and obtain

$$(7C+2D)\cos(2t) + (-2C+7D)\sin(2t) = 5\cos(2t)$$

Equating like coefficients,

$$7C + 2D = 5$$
$$-2C + 7D = 0$$

From which,  $C = \frac{35}{53}$  and  $D = \frac{10}{53}$ .

The characteristic polynomial is

$$M + 7 = 0$$

Thus, the total solution is

$$x(t) = A e^{-7t} + \left(\frac{35}{53}\cos[2t] + \frac{10}{53}\sin[2t]\right)$$

Solving for the arbitrary constants,  $x(0) = A + \frac{35}{53} = 0$ . Therefore,  $A = -\frac{35}{53}$ . The final solution is

$$x(t) = \left(-\frac{35}{53}\right)e^{-7t} + \left(\frac{35}{53}\cos[2t] + \frac{10}{53}\sin[2t]\right)$$

b. Assume a particular solution of

$$x_p = Asin3t + Bcos3t$$

Substitute into the differential equation and obtain

$$(18A - B)\cos(3t) - (A + 18B)\sin(3t) = 5\sin(3t)$$

Therefore, 18A - B = 0 and -(A + 18B) = 5. Solving for A and B we obtain

$$x_p = (-1/65)\sin 3t + (-18/65)\cos 3t$$

The characteristic polynomial is

$$M^{2}+6M+8=(M+4)(M+2)$$

Thus, the total solution is

x = Ce<sup>-4t</sup> + De<sup>-2t</sup> + 
$$\left(-\frac{18}{65}\cos(3t) - \frac{1}{65}\sin(3t)\right)$$

Solving for the arbitrary constants,  $x(0) = C + D - \frac{18}{65} = 0$ .

Also, the derivative of the solution is

$$\frac{dx}{dt} = -\frac{3}{65}\cos(3t) + \frac{54}{65}\sin(3t) - 4Ce^{-4t} - 2De^{-2t}$$

Solving for the arbitrary constants,  $\dot{x}(0) - \frac{3}{65} - 4C - 2D = 0$ , or  $C = -\frac{3}{10}$  and  $D = \frac{15}{26}$ .

The final solution is

$$x = -\frac{18}{65}\cos(3t) - \frac{1}{65}\sin(3t) - \frac{3}{10}e^{-4t} + \frac{15}{26}e^{-2t}$$

c. Assume a particular solution of

$$x_p = A$$

Substitute into the differential equation and obtain 25A = 10, or A = 2/5.

The characteristic polynomial is

$$M^{2} + 8M + 25 = (M + 4 + 3i)(M + 4 - 3i)$$

Thus, the total solution is

$$x = \frac{2}{5} + e^{-4t} (B \sin(3t) + C \cos(3t))$$

Solving for the arbitrary constants, x(0) = C + 2/5 = 0. Therefore, C = -2/5. Also, the derivative of the solution is

$$\frac{dx}{dt} = ((3 B-4 C) \cos(3 t) - (4 B+3 C) \sin(3 t)) e^{-4t}$$

Solving for the arbitrary constants,  $\dot{x}(0) = 3B - 4C = 0$ . Therefore, B = -8/15. The final solution is

$$x(t) = \frac{2}{5} - e^{-4t} \left( \frac{8}{15} \sin(3t) + \frac{2}{5} \cos(3t) \right)$$

21.

a. Assume a particular solution of

$$x_p(t) = C\cos(2t) + D\sin(2t)$$

Substitute into the differential equation and obtain

$$-2(C-2D)\cos(2t) - 4\left(C + \frac{1}{2}D\right)\sin(2t) = \sin(2t)$$

Equating like coefficients,

$$-2(C-2D) = 0$$
$$-4\left(C + \frac{1}{2}D\right) = 1$$

From which,  $C = -\frac{1}{5}$  and  $D = -\frac{1}{10}$ .

The characteristic polynomial is

$$M^{2} + 2M + 2 = (M + 1 + i)(M + 1 - i)$$

Thus, the total solution is

$$x = -\frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + e^{-t}(A\cos[t] + B\sin[t])$$

Solving for the arbitrary constants,  $x(0) = A - \frac{1}{5} = 2$ . Therefore,  $A = \frac{11}{5}$ . Also, the derivative of the

solution is

$$\frac{\mathrm{dx}}{\mathrm{dt}} = -\frac{1}{5}\cos(2t) + \frac{2}{5}\sin(2t) + (-A+B)e^{-t}\cos(t) - (A+B)e^{-t}\sin(t)$$

Solving for the arbitrary constants,  $\dot{x}(0) = -A + B - 0.2 = -3$ . Therefore,  $B = -\frac{3}{5}$ . The final solution

is

$$x(t) = -\frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + e^{-t}\left(\frac{11}{5}\cos(t) - \frac{3}{5}\sin(t)\right)$$

**b.** Assume a particular solution of

$$x_p = Ce^{-2t} + Dt + E$$

Substitute into the differential equation and obtain

$$C e^{-2t} + Dt + 2D + E = 5e^{-2t} + t$$

Equating like coefficients, C = 5, D = 1, and 2D + E = 0.

From which, C = 5, D = 1, and E = -2.

The characteristic polynomial is

$$M^{2} + 2M + 1 = (M + 1)^{2}$$

Thus, the total solution is

$$x(t) = A e^{-t} + B e^{-t} t + 5e^{-2t} + t - 2$$

Solving for the arbitrary constants, x(0) = A + 5 - 2 = 2 Therefore, A = -1. Also, the derivative of the solution is

$$\frac{dx}{dt} = (-A + B)e^{-t} - Bte^{-t} - 10e^{-2t} + 1$$

Solving for the arbitrary constants, x(0) = B - 8 = 1. Therefore, B = 9. The final solution is

$$x(t) = -e^{-t} + 9t e^{-t} + 5e^{-2t} + t - 2$$

c. Assume a particular solution of

$$x_p = Ct^2 + Dt + E$$

Substitute into the differential equation and obtain

$$4Ct^{2} + 4Dt + 2C + 4E = t^{2}$$

Equating like coefficients,  $C = \frac{1}{4}$ , D = 0, and 2C + 4E = 0.

From which,  $C = \frac{1}{4}$ , D = 0, and  $E = -\frac{1}{8}$ .

The characteristic polynomial is

$$M^{2} + 4 = (M + 2i)(M - 2i)$$

Thus, the total solution is

$$x(t) = A\cos(2t) + B\sin(2t) + \frac{1}{4}t^2 - \frac{1}{8}$$
  
Solving for the arbitrary constants,  $x(0) = A - \frac{1}{8} = 1$  Therefore,  $A = \frac{9}{8}$ . Also, the derivative of the

solution is

$$\frac{\mathrm{dx}}{\mathrm{dt}} = 2B\cos(2t) - 2A\sin(2t) + \frac{1}{2}t$$

Solving for the arbitrary constants, x(0) = 2B = 2. Therefore, B = 1. The final solution is

$$x(t) = \frac{9}{8}\cos(2t) + \sin(2t) + \frac{1}{4}t^2 - \frac{1}{8}$$











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