

Chapter One: Management Science

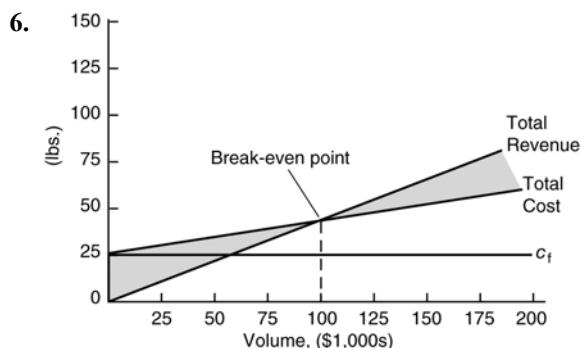
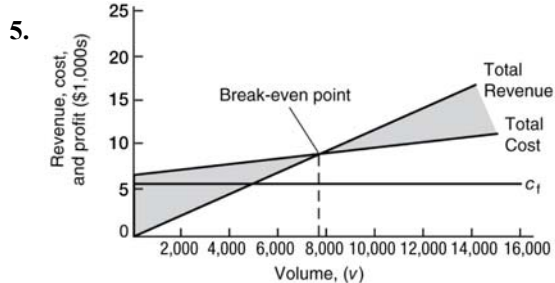
PROBLEM SUMMARY

1. Total cost, revenue, profit, and break-even
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4. Break-even volume
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6. Graphical analysis (1-4)
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36. Linear programming
37. Linear programming
38. Linear programming
39. Forecasting/statistics
40. Linear programming
41. Waiting lines
42. Shortest route

PROBLEM SOLUTIONS

1. a) $v = 300$, $c_f = \$8,000$,
 $c_v = \$65$ per table, $p = \$180$;
 $TC = c_f + vc_v = \$8,000 + (300)(65) = \$27,500$;
 $TR = vp = (300)(180) = \$54,000$;
 $Z = \$54,000 - 27,500 = \$26,500$ per month
- b) $v = \frac{c_f}{p - c_v} = \frac{8,000}{180 - 65} = 69.56$ tables per month
2. a) $v = 12,000$, $c_f = \$18,000$, $c_v = \$0.90$,
 $p = \$3.20$;
 $TC = c_f + vc_v$
 $= 18,000 + (12,000)(0.90)$
 $= \$28,800$;
 $TR = vp = (12,000)(\$3.20) = \$38,400$;
 $Z = \$38,400 - 28,800 = \$9,600$ per year
- b) $v = \frac{c_f}{p - c_v} = \frac{18,000}{3.20 - 0.90} = 7,826$
3. a) $v = 18,000$, $c_f = \$21,000$, $c_v = \$0.45$,
 $p = \$1.30$;
 $TC = c_f + vc_v = \$21,000 + (18,000)(.45) = \$29,100$;
 $TR = vp = (18,000)(1.30) = \$23,400$;
 $Z = \$23,400 - 29,100 = -\$5,700$ (loss)
- b) $v = \frac{c_f}{p - c_v} = \frac{21,000}{1.30 - .45} = 24,705.88$ yd per month
4. $c_f = \$25,000$, $p = \$0.40$, $c_v = \$0.15$,
 $v = \frac{c_f}{p - c_v} = \frac{25,000}{.40 - .15} = 100,000$ lb per month



7.
$$v = \frac{c_f}{p - c_v} = \frac{\$25,000}{30 - 10} = 1,250 \text{ dolls}$$

8. Break-even volume as percentage of capacity

$$= \frac{v}{k} = \frac{7,826}{12,000} = .652 = 65.2\%$$

9. Break-even volume as percentage of capacity

$$= \frac{v}{k} = \frac{24,750.88}{25,000} = .988 = 98.8\%$$

10. Break-even volume as percentage of capacity

$$\text{capacity} = \frac{v}{k} = \frac{100,000}{120,000} = .833 = 83.3\%$$

11.
$$v = \frac{c_f}{p - c_v} = \frac{18,000}{2.75 - 0.90} = 9,729.7 \text{ cupcakes}$$

It increases the break-even volume from 7,826 to 9,729.7 per year.

12.
$$v = \frac{c_f}{p - c_v} = \frac{25,000}{.60 - .15} = 55,555.55 \text{ lb}$$

per month; it reduces the break-even volume from 100,000 lb per month to 55,555.55 lb.

13.
$$v = \frac{c_f}{p - c_v} = \frac{25,000}{.60 - .22} = 65,789.47 \text{ lb}$$

per month; it increases the break-even volume from 55,555.55 lb per month to 65,789.47 lb per month.

14.
$$v = \frac{c_f}{p - c_v} = \frac{39,000}{.60 - .22} = 102,613.57 \text{ lb}$$

per month; it increases the break-even volume from 65,789.47 lb per month to 102,613.57 lb per month.

15. Initial profit: $Z = vp - c_f - vc_v = (9,000)(.75) - 4,000 - (9,000)(.21) = 6,750 - 4,000 - 1,890 = \860 per month; increase in price: $Z = vp - c_f - vc_v = (5,700)(.95) - 4,000 - (5,700)(.21) = 5,415 - 4,000 - 1,197 = \218 per month; the dairy should not raise its price.

16. a)
$$v = \frac{c_f}{p - c_v} = \frac{30,000}{30 - 15} = 2,000$$

b)
$$v = \frac{\text{Ad expenses} + c_f}{p - c_v} = \frac{39,000}{30 - 15} = 2,600$$

The increase in fixed cost from \$30,000 to \$39,000 will increase the break-even from 2,000 toys to 2,600 toys, an increase of 600 toys; however, David can only sell 500 more toys. Thus, he should not spend the extra \$9,000 on advertising.

17. Original break-even point (from 16a) is 2,000 toys.
New break-even point:

$$v = \frac{c_f}{p - c_v} = \frac{20,000}{30 - 20} = 2,000$$

The break-even point remains the same.

18. a)
$$v = \frac{c_f}{p - c_v} = \frac{\$27,000}{8.95 - 3.75} = 5,192.30 \text{ pizzas}$$

b)
$$\frac{5,192.3}{20} = 259.6 \text{ days}$$

c) Revenue for the first 30 days = $30(pv - vc_v)$

$$= 30[(8.95)(20) - (20)(3.75)]$$

$$= \$3,120$$

$\$27,000 - 3,120 = \$23,880$, portion of fixed cost not recouped after 30 days.

$$\text{New } v = \frac{c_f}{p - c_v} = \frac{\$23,880}{7.95 - 3.75} = 5,685.7 \text{ pizzas}$$

$$\text{Total break-even volume} = 600 + 5,685.7 = 6,285.7 \text{ pizzas}$$

$$\begin{aligned} \text{Total time to break-even} &= 30 + \frac{5,685.7}{20} \\ &= 314.3 \text{ days} \end{aligned}$$

19. a) Cost of Regular plan = $\$55 + (.33)(260 \text{ minutes})$
= \$140.80

Cost of Executive plan = $\$100 + (.25)(60 \text{ minutes})$
= \$115

Select Executive plan.

b) $55 + (x - 1,000)(.33) = 100 + (x - 1,200)(.25)$
 $- 275 + .33x = .25x - 200$
 $x = 937.50 \text{ minutes per month or } 15.63 \text{ hrs.}$

20. $c_f = \$26,000$
 $c_v = \$0.67 (\$5.36/8 = 0.67)$
 $p = \$3.75$

$$v = \frac{26,000}{3.75 - 0.67}$$

$$= 8,442 \text{ slices}$$

$$\text{Forecasted annual demand} = (540)(52) = 28,080$$

$$Z = \$91,260 - 44,813.6 = 46,446.4$$

21.

OLD

New

$$\begin{aligned} 26,000 + (.67)v &= 30,000 + (.48)v \\ .19v &= 4,000 \\ v &= 21,053 \text{ slices} \end{aligned}$$

$$Z = \text{New profit} - \text{old profit}$$

$$Z = \$47,781.60 - 46,446.40$$

$$= \$1,335.20$$

Purchase equipment

22. a) $14,000 = \frac{7,500}{p - .35}$

$$p = \$0.89 \text{ to break even}$$

b) If the team did not perform as well as expected the crowds could be smaller; bad weather could reduce crowds and/or affect what fans eat at the game; the price she charges could affect demand.

c) This will be a subjective answer, but \$1.25 seems to be a reasonable price.

$$Z = vp - c_f - vc_v$$

$$\begin{aligned} Z &= (14,000)(1.25) - 7,500 - (14,000)(0.35) \\ &= 17,500 - 12,400 \\ &= \$5,100 \end{aligned}$$

23. a) $c_f = \$1,700$

$$c_v = \$12 \text{ per pupil}$$

$$p = \$75$$

$$v = \frac{1,700}{75 - 12}$$

$$= 26.98 \text{ or } 27 \text{ pupils}$$

b) $Z = vp - c_f - vc_v$

$$\$5,000 = v(75) - \$1,700 - v(12)$$

$$63v = 6,700$$

$$v = 106.3 \text{ pupils}$$

c) $Z = vp - c_f - vc_v$

$$\$5,000 = 60p - \$1,700 - 60(12)$$

$$60p = 7,420$$

$$p = \$123.67$$

24. a) $c_f = \$350,000$

$$c_v = \$12,000$$

$$p = \$18,000$$

$$v = \frac{c_f}{p - c_v}$$

$$= \frac{350,000}{18,000 - 12,000}$$

$$= 58.33 \text{ or } 59 \text{ students}$$

b) $Z = (75)(18,000) - 350,000 - (75)(12,000)$
= \$100,000

c) $Z = (35)(25,000) - 350,000 - (35)(12,000)$
= 105,000

This is approximately the same as the profit for 75 students and a lower tuition in part (b).

25. $p = \$300$

$$c_f = \$10,000$$

$$c_v = \$70$$

$$Z = \$75,000$$

$$v = \frac{Z + c_f}{p - c_v} = \frac{75,000 + 10,000}{300 - 70} = 369.57 \text{ teams}$$

26. Fixed cost (c_f) = \$875,000

$$\text{Variable cost } (c_v) = \$200$$

$$\text{Price } (p) = (225)(12) = \$2,700$$

$$\begin{aligned} v &= c_f / (p - c_v) = 875,000 / (2,700 - 200) \\ &= 350 \end{aligned}$$

With volume doubled to 700:

$$\text{Profit } (Z) = (2,700)(700) - 875,000 - (700)(200) \\ = \$875,000$$

27. Fixed cost (c_f) = 100,000
 Variable cost (c_v) = $\$.50(.35) + (.35)(.50) + (.15)(2.30)$
 $= \$0.695$
 Price (p) = \$6
 Profit (Z) = $(6)(45,000) - 100,000 - (45,000)(0.695)$
 $= \$138,725$

This is not the financial profit goal of \$150,000.

The price to achieve the goal of \$150,000 is,

$$p = (Z + c_f + vc_v)/v \\ = (150,000 + 100,000 + (45,000)(.695))/45,000 \\ = \$6.25$$

The volume to achieve the goal of \$150,000 is,

$$v = (Z + c_f)/(p - c_v) \\ = (150,000 + 100,000)/(6 - .695) \\ = 47,125$$

28. a) Monthly fixed cost (c_f) = cost of van/60 months
 + labor (driver)/month
 $= (21,500/60) + (30.42$
 $\text{days/month})(\$8/\text{hr})$
 (5 hr/day)
 $= 358.33 + 1,216.80$
 $= \$1,575.13$

$$\text{Variable cost } (c_v) = \$1.35 + 15.00 \\ = \$16.35$$

$$\text{Price } (p) = \$34 \\ v = c_f/(p - v_c) \\ = (1,575.13)/(34 - 16.35) \\ v = 89.24 \text{ orders/month}$$

- b) $89.24/30.42 = 2.93$ orders/day – Monday through Thursday

Double for weekend = 5.86 orders/day – Friday through Sunday

$$\text{Orders per month} = \text{approximately } (18 \text{ days}) \\ (2.93 \text{ orders}) + (12.4 \text{ days})(5.86 \text{ orders}) \\ = 125.4 \text{ delivery orders per month}$$

$$\text{Profit} = \text{total revenue} - \text{total cost} \\ = vp - (c_f + vc_v) \\ = (125.4)(34) - 1,575.13 - (125.4)(16.35) \\ = 638.18$$

29. a) $v = \frac{c_f}{p - c_v} = \frac{500}{30 - 14}$
 $v = 31.25$ jobs

b) $(8 \text{ weeks})(6 \text{ days/week})(3 \text{ lawns/day}) = 144$
 lawns
 $Z = (144)(30) - 500 - (144)(14)$
 $Z = \$1,804$

c) $(8 \text{ weeks})(6 \text{ days/week})(4 \text{ lawns/day}) = 192$ lawns
 $Z = (192)(25) - 500 - (192)(14)$
 $Z = \$1,612$

No, she would make less money than (b)

30. a) $v = \frac{c_f}{p - c_v} = \frac{500}{10 - 2} = 62.50$
 $v = 62.50$ cars

b) $(5 \text{ weeks})(3 \text{ days/week})(10 \text{ cars/day}) = 150$ cars
 $Z = (150)(10) - 500 - (150)(2)$
 $Z = \$700$

c) $(5 \text{ weeks})(3 \text{ days/week})(4 \text{ lots/day}) = 60$ lots
 $Z = (60)(125) - 1,500 - (60)(25)$
 $Z = \$4,500$

Yes, better than (b).

31. $c_f = \$7,500$
 Monthly $c_f = (\$2,300)(12)$
 $= \$27,600$
 Total $c_f = \$35,100$
 $c_v = 0$
 $p = \$0.24$

$$v = \frac{c_f}{p} = \frac{35,100}{.24} = 146,250 \text{ hits per year}$$

$$v = 12,188 \text{ hits per month}$$

$$\$45,000 = v(.24) - (12)(3,500) - (0)v$$

$$.24v = 87,000$$

$$v = 362,500$$

$$v = 30,208 \text{ hits per month}$$

32. a) This is a “multiproduct” break-even problem.
 The formula for the break-even volume is

$$v = \frac{\text{Total fixed cost}}{\left(\frac{\text{weighted average}}{\text{selling price}} \right) - \left(\frac{\text{weighted average}}{\text{variable cost}} \right)}$$

$$v = \frac{20,000}{[(3.50)(0.75) + (2.50)(0.25)] - [(1)(0.75) + (0.50)(0.25)]}$$

$$v = 8,421.05 \text{ units}$$

$$\text{Cupcakes} = (8,421.05)(0.75) = 6,315.79$$

$$\text{Chocolate cookies} = (8,421.05)(0.25) \\ = 2,105.26$$

b) Break-even sales in dollars = $(6,315.79)(3.50) + (2,105.26)(2.50) = \$27,368.42$

33. This is a decision analysis problem – the subject of Chapter 12. The payoff table is:

Decision Alternatives	Weather Conditions	
	Good	Bad
\$3.25	\$12,800	\$8,450
\$4.00	\$14,400	\$5,275

The student’s decision depends on the degree of risk they are willing to assume.

Chapter 12 includes decision criteria for this problem.

34. This problem uses expected value for the decision alternatives in problem 30.

Expected value $(\$3.25) = (\$12,800)(0.60) + (\$8,450)(0.40) = \$11,060$

Expected value $(\$4.00) = (\$14,400)(0.60) + (\$5,275)(0.40) = \$10,750$

Although the decision to sell hotdogs for \$3.25 results in the greatest expected value, the results are so close, Annie would likely be indifferent.

35. There are two possible answers, or solution points: $x = 25, y = 0$ or $x = 0, y = 50$

Substituting these values in the objective function:

$Z = 15(25) + 10(0) = 375$

$Z = 15(0) + 10(50) = 500$

Thus, the solution is $x = 0$ and $y = 50$

This is a simple linear programming model, the subject of the next several chapters. The student should recognize that there are only two possible solutions, which are the corner points of the feasible solution space, only one of which is optimal.

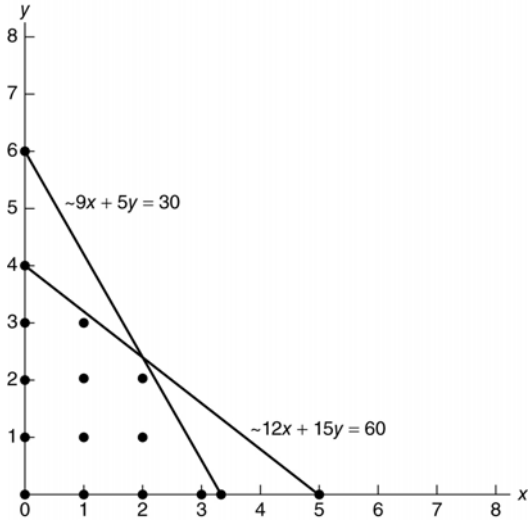
36. The solution is computed by solving simultaneous equations,

$x = 30, y = 10, Z = \$1,400$

It is the only, i.e., “optimal” solution because there is only one set of values for x and y that satisfy both constraints simultaneously.

- 37.

# bowls	# mugs	Labor usage $12x + 15y \leq 60$	Clay usage $9x + 5y \leq 30$	Profit $300x + 250y$	Possible solution?
0	1	15	5	250	yes
1	0	12	9	300	yes
1	1	27	14	550	yes
0	2	30	10	500	yes
2	0	24	18	600	yes
1	2	42	19	800	yes
2	1	39	23	850	yes
2	2	54	28	1100	yes, best solution
0	3	45	15	750	yes
3	0	36	27	900	yes
1	3	57	24	1050	yes
3	1	51	32	1150	no
2	3	69	33	1350	no
3	2	66	37	1400	no
3	3	81	42	1650	no
4	0	48	36	1200	no
0	4	60	20	1000	yes
1	4	72	29	1300	no
4	1	63	41	1450	no
2	4	84	38	1600	no
4	2	78	46	1700	no



38. Minimize $Z = 40X_{MH} + 80X_{MB} + 50X_{CH} + 70X_{CB}$
 Subject to
 $X_{MH} + X_{MB} = 500$
 $X_{CH} + X_{CB} = 500$
 $X_{MH} + X_{CH} = 600$
 $X_{MB} + X_{CB} = 400$
 The solution is $X_{MH} = 500, X_{CH} = 100, X_{CB} = 400,$
 $Z = \$53,000$
 The problem can be solved by allocating as much as possible to the lowest cost variable $X_{MH} = 500$, then repeating this step until all the demand has been met. This is a similar logic to the minimum cell cost method.

39. This is virtually a straight linear relationship between time and site visits; thus, a simple linear graph would result in a forecast of approximately 34,500 site visits.
40. Determine logical solutions:

	Cakes	Bread	Total Sales
1.	0	2	\$12
2.	1	2	\$22
3.	3	1	\$36
4.	4	0	\$40

Each solution must be checked to see if it violates the constraints for baking time and flour. Some possible solutions can be logically discarded because they are obviously inferior. For example, 0 cakes and 1 loaf of bread is clearly inferior to 0 cakes and 2 loaves of bread. 0 cakes and 3 loaves of bread is not possible because there is not enough flour for 3 loaves of bread.

Using this logic, there are four possible solutions as shown. The best one, 4 cakes and 0 loaves of bread, results in the highest total sales of \$40.

41. This problem demonstrates the cost trade-off inherent in queuing analysis, the topic of Chapter 13. In this problem the cost of service, i.e., the cost of staffing registers, is added to the cost of customers waiting, i.e., the cost of lost sales and ill will, as shown in the following table.

Registers staffed	1	2	3	4	5	6	7	8
Waiting time (mins)	20	14	9	4	1.7	1	0.5	0.1
Cost of service (\$)	60	120	180	240	300	360	420	480
Cost of waiting (\$)	850	550	300	50	0	0	0	0
Total cost (\$)	910	670	480	290	300	360	420	480

The total minimum cost of \$290 occurs with 4 registers staffed

42. The shortest route problem is one of the topics of Chapter 7. At this point, the most logical “trial and error” way that most students will probably approach this problem is to identify all the feasible routes and compute the total distance for each, as follows:
- 1-2-6-9 = 228
 1-2-5-9 = 213
 1-3-5-9 = 211
 1-3-8-9 = 276
 1-4-7-8-9 = 275
- Obviously inferior routes like 1-3-4-7-8-9 and 1-2-5-8-9 that include additional segments to the routes listed above can be logically eliminated

from consideration. As a result, the route 1-3-5-9 is the shortest.

An additional aspect to this problem could be to have the students look at these routes on a real map and indicate which they think might “practically” be the best route. In this case, 1-2-5-9 would likely be a better route, because even though it’s two miles farther it is Interstate highway the whole way, whereas 1-3-5-9 encompasses U.S. 4-lane highways and state roads.

CASE SOLUTION: CLEAN CLOTHES CORNER LAUNDRY

a) $v = \frac{c_f}{p - c_v} = \frac{1,700}{1.10 - .25} = 2,000$ items per month

- b) Solution depends on number of months; 36 used here. $\$16,200 \div 36 = \450 per month, thus monthly fixed cost is $\$2,150$

$$v = \frac{c_f}{p - c_v} = \frac{2,150}{1.10 - .25} = 2,529.4 \text{ items per month}$$

529.4 additional items per month

c) $Z = vp - c_f - vc_v$
 $= 4,300(1.10) - 2,150 - 4,300(.25)$
 $= \$1,505$ per month

After 3 years, $Z = \$1,955$ per month

d) $v = \frac{c_f}{p - c_v} = \frac{1,700}{.99 - .25} = 2,297.3$

$$Z = vp - c_f - vc_v$$

$$= 3,800(.99) - 1,700 - 3,800(.25)$$

$$= \$1,112 \text{ per month}$$

- e) With both options:

$$Z = vp - c_f - vc_v$$

$$= 4,700(.99) - 2,150 - 4,700(.25)$$

$$= \$1,328$$

She should purchase the new equipment but not decrease prices.

CASE SOLUTION: OCOBEE RIVER RAFTING COMPANY

Alternative 1: $c_f = \$3,000$

$$p = \$20$$

$$c_v = \$12$$

$$v_1 = \frac{c_f}{p - c_v} = \frac{3,000}{20 - 12} = 375 \text{ rafts}$$

Alternative 2: $c_f = \$10,000$

$$p = \$20$$

$$c_v = \$8$$

$$v_2 = \frac{c_f}{p - c_v} = \frac{10,000}{20 - 8} = 833.37$$

If demand is less than 375 rafts, the students should not start the business.

If demand is less than 833 rafts, alternative 2 should not be selected, and alternative 1 should be used if demand is expected to be between 375 and 833.33 rafts.

If demand is greater than 833.33 rafts, which alternative is best? To determine the answer, equate the two cost functions.

$$3,000 + 12v = 10,000 + 8v$$

$$4v = 7,000$$

$$v = 1,750$$

This is referred to as the point of indifference between the two alternatives. In general, for demand lower than this point (1,750) the alternative should be selected with the lowest fixed cost; for demand greater than this point the alternative with the lowest variable cost should be selected. (This general relationship can be observed by graphing the two cost equations and seeing where they intersect.)

Thus, for the Ocobee River Rafting Company, the following guidelines should be used:

demand < 375 , do not start business; $375 < \text{demand} < 1,750$, select alternative 1; demand $> 1,750$, select alternative 2

Since Penny estimates demand will be approximately 1,000 rafts, alternative 1 should be selected.

$$Z = v_p - c_f - vc_v$$

$$= (1,000)(20) - 3,000 - (1,000)(12)$$

$$Z = \$5,000$$

CASE SOLUTION: CONSTRUCTING A DOWNTOWN PARKING LOT IN DRAPER

- a) The annual capital recovery payment for a capital expenditure of \$4.5 million over 30 years at 8% is,

$$(4,500,000)[0.08(1 + .08)^{30}] / (1 + .08)^{30} - 1$$

$$= \$399,723.45$$

This is part of the annual fixed cost. The other part of the fixed cost is the employee annual salaries of \$140,000. Thus, total fixed costs are,

$$\$399,723.45 + 140,000 = \$539,723.45$$

$$v = \frac{c_f}{p - c_v}$$

$$= \frac{539,723.45}{3.20 - 0.60}$$

$$= 207,585.94 \text{ parked cars per year}$$

b) If 365 days per year are used, then the daily usage is,

$$\frac{207,585.94}{365} = 568.7 \text{ or approximately } 569 \text{ cars per day}$$

This seems like a reachable goal given the size of the town and the student population.

CASE SOLUTION: A BUS SERVICE FOR DRAPER

Fixed cost (3 buses) = 1,200,000
Total Variable Cost = 591,300
Annual Revenue = 648,240

Passengers/bus/trip = 37
Passenger fare = 4
Trips/bus/day = 4
Number of buses = 3
Days/year = 365
Total annual revenue = 648,240 = (37)(4)(4)(3)(365)

Bus operating hrs/day = 18
Operating cost/hr = 90
Days/year = 365
Total annual variable cost = 591,300 = (18)(90)(365)

(a) First year loss = (1,143,060.00)

(b) Years to break even:

Loss in year 1 = -1,143,060.0

Not possible to break even

(c) 45 passengers per trip:

Annual Revenue = 788,400

First year loss = (1,002,900)

Not possible to break even

50 passengers per trip:

Annual revenue = 876,000

First year loss = (915,300)

Break even year: (3.215) years

(d) Decrease in trips:

Annual revenue = 657,000

Total variable cost = 443,475

First year loss = (986,475)

Break even year: (5.62) years

Bus operating hrs/day = 13.5

Operating cost/hr. = 90

Days/year = 365

Total annual variable cost = 443,475

(e) \$1,200,000 Grant:

Fixed Cost = 0

First Year Revenue = 56,940