Chapter 1

Functions

1.1 Functions and Their Graphs

6. $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ does not define y as a function of x since its graph fails the vertical line test.

8. $x = y^3$ defines y as a function of x since its graph satisfies the vertical line test.

10. $|x| - |y| = 0$ does not define y as a function of x since its graph fails the vertical line test.

12. a. Substituting 1 for x in the formula for $g(x)$ gives

$$\begin{aligned} g(1) &= (1)^3 - 4(1) \\ &= 1 - 4 \\ &= -3 \end{aligned}$$

b. Substituting -2 for x in the formula for $g(x)$ gives

$$\begin{aligned} g(-2) &= (-2)^3 - 4(-2) \\ &= -8 - (-8) \\ &= 0 \end{aligned}$$

c. Substituting 3 for x in the formula for $g(x)$ gives

$$\begin{aligned} g(3) &= (3)^3 - 4(3) \\ &= 27 - 12 \\ &= 15 \end{aligned}$$

14. a. Substituting 2 in for y in the formula for $h(y)$ gives

$$\begin{aligned} h(2) &= \frac{(2)^2 - 3}{2 + 7} \\ &= \frac{4 - 3}{2 + 7} \\ &= \frac{1}{9} \end{aligned}$$

b. Substituting -6 in for y in the formula for $h(y)$ gives

$$\begin{aligned} h(-6) &= \frac{(-6)^2 - 3}{(-6) + 7} \\ &= \frac{36 - 3}{(-6) + 7} \\ &= 33 \end{aligned}$$

c. Substituting x in for y in the formula for $h(y)$ gives

$$h(x) = \frac{x^2-3}{x+7}$$

16. a. Substituting t in for x in the formula for $f(x)$ gives

$$f(t) = \frac{2t+2}{2t-3}$$

b. Substituting $x-1$ in for x in the formula for $f(x)$ gives

$$\begin{aligned} f(x-1) &= \frac{2(x-1)+2}{2(x-1)-3} \\ &= \frac{2x-2+2}{2x-2-3} \\ &= \frac{2x}{2x-5} \end{aligned}$$

c. Substituting $\frac{x}{2}$ in for x in the formula for $f(x)$ gives

$$\begin{aligned} f\left(\frac{x}{2}\right) &= \frac{2(\frac{x}{2})+2}{2(\frac{x}{2})-3} \\ &= \frac{x+2}{x-3} \end{aligned}$$

18. a. Substituting $\square$ for x in the formula for $p(x)$ gives

$$p(\square) = \sqrt{9-\square^2}$$

b. Substituting $x+3$ for x in the formula for $p(x)$ gives

$$\begin{aligned} p(x+3) &= \sqrt{9-(x+3)^2} \\ &= \sqrt{9-(x^2+6x+9)} \\ &= \sqrt{9-x^2-6x-9} \\ &= \sqrt{-x^2-6x} \end{aligned}$$

c. Substituting $x-3$ for x in the formula for $p(x)$ gives

$$\begin{aligned} p(x-3) &= \sqrt{9-(x-3)^2} \\ &= \sqrt{9-(x^2-6x+9)} \\ &= \sqrt{9-x^2+6x-9} \\ &= \sqrt{-x^2+6x} \end{aligned}$$

20. a. Substituting $g(2) = 2(2)+1 = 5$ for x in the formula for $f(x)$ gives

$$\begin{aligned} f(g(2)) &= f(5) \\ &= 5^2 \\ &= 25 \end{aligned}$$

b. Substituting $g(t) = 2t + 1$ for x in the formula for $f(x)$ gives

$$\begin{aligned} f(g(t)) &= f(2t+1) \\ &= (2t+1)^2 \end{aligned}$$

c. First, we substitute n^2 in for x in the formula for $f(x)$

$$\begin{aligned} f(n^2) &= (n^2)^2 \\ &= n^4 \end{aligned}$$

Then, substituting $f(n^2) = n^4$ in for y in the formula for $g(y)$ gives

$$\begin{aligned} g(f(n^2)) &= g\left(n^4\right) \\ &= 2n^4 + 1 \end{aligned}$$

22. If we denote the input by x, then the cube of the input is x^3 and twice the cube of the input is $2x^3$. Thus, $g(x) = 2x^3$.

24. If we denote the input by x, then the square root of the input is $\sqrt{x}$ and the reciprocal of the square root of the input is $\dfrac{1}{\sqrt{x}}$. Thus, $p(x) = \dfrac{1}{\sqrt{x}}$.

26. The domain contains all the values of x for which the function has an output value. For this function, the only way the function will not have an output value is if the denominator equals 0, which happens when $x = 0$. Therefore, the domain of the function $g(x) = \frac{4}{x}$ is $\{x \mid x \neq 0\}$.

28. The absolute value function is defined for all real numbers. Thus, the domain of $r(s) = |s|$ is $(-\infty, \infty)$.

30. Since we are only considering the input values for which the function is defined, the domain includes all values of x except those for which the denominator is 0. To determine the values that must be excluded, we set $x^2 - 5x + 6 = 0$ and solve for x, as follows.

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x-2)(x-3) &= 0 \\ x = 2,\ x &= 3 \end{aligned}$$

Thus, the domain of p is $\{x|x \neq 2, x \neq 3\}$.

32. Since we are only considering values of t for which $g(t)$ is real, the valid input values of t are those for which $t - 3 \geq 0$. Solving for t gives $t \geq 3$. Therefore, the domain of g is $[3, \infty)$.

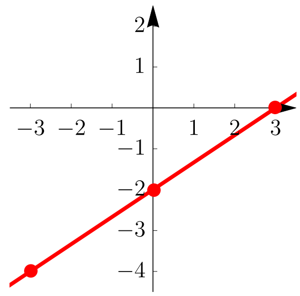
34. The domain is the set of all y values for which the function is defined. Consequently, y must be a value for which the expression under the radical is not negative and for which the denominator is not zero. Any other values are acceptable. Thus, we must require that $2y - 6 > 0$, which is equivalent to $y > 3$. Thus, the domain is $\{y|y > 3\}$.

36. The domain contains all the values of x for which the function has an output value. For this function, the only way the function will not have an output value is if the denominator equals 0, which happens when $x = 0$. Therefore, the domain of the function $g(x) = \frac{x}{x}$ is $\{x|x \neq 0\}$.

38. We first complete the table by computing values of $f(x)$ for the given values of x.

$\boldsymbol{x}$	$\boldsymbol{f(x) = \frac{2}{3}x - 2}$
-3	$f(-3) = \frac{2}{3}(-3) - 2 = -2 - 2 = -4$
0	$f(0) = \frac{2}{3}(0) - 2 = 0 - 2 = -2$
3	$f(3) = \frac{2}{3}(3) - 2 = 2 - 2 = 0$

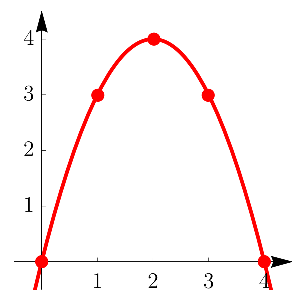
We now plot the points $(-3, -4)$, $(0, -2)$, and $(3, 0)$, and connect them with a smooth curve. The resulting graph is a line and is shown below.



40. We begin by computing $f(x)$ for the given values of x.

x	$f(x) = -(x-2)^2 + 4$
0	$f(0) = -(0-2)^2 + 4 = -(-2)^2 + 4 = -4 + 4 = 0$
1	$f(1) = -(1-2)^2 + 4 = -(-1)^2 + 4 = -1 + 4 = 3$
2	$f(2) = -(2-2)^2 + 4 = -0^2 + 4 = 0 + 4 = 4$
3	$f(3) = -(3-2)^2 + 4 = -1^2 + 4 = -1 + 4 = 3$
4	$f(4) = -(4-2)^2 + 4 = -2^2 + 4 = -4 + 4 = 0$

We now plot the points $(0,0)$, $(1,3)$, $(2,4)$, $(3,3)$, and $(4,0)$, and connect them with a smooth line. The resulting graph is a parabola and is shown below.



42. Since $-4 < 0$, $g(-4)$ is defined by the first piece of the function.

$$g(-4) = 3(-4) + 2 = -10$$

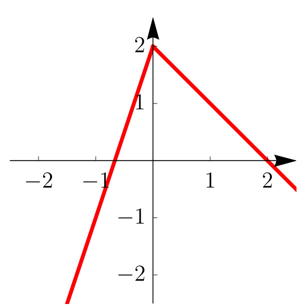
$g(0)$ is defined by the second piece of the function.

$$g(0) = -(0) + 2 = 2$$

$g(1)$ is defined by the second piece of the function.

$$g(1) = -(1) + 2 = 1$$

To sketch the graph, we plot the line $y = 3x + 2$ for $x < 0$ and the line $y = -x + 2$ for $x \geq 0$. The resulting graph is shown below.



44. Since $-3 < -2$, $h(-3)$ is defined by the first piece of the function.

$$h(-3) = -(-3) - 1 = 2$$

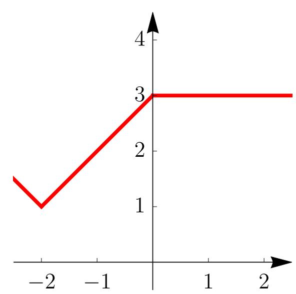
$h(0)$ is defined by the second piece of the function.

$$h(0) = 0 + 3 = 3$$

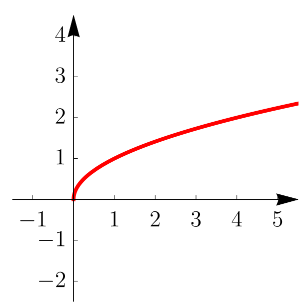
$h(6)$ is defined by the third piece of the function.

$$h(6) = 3$$

To sketch the graph, we plot the line $y = -x - 1$ for $x < -2$, the line $y = x + 3$ for $-2 \leq x \leq 0$, and the line $y = 3$ for $x > 0$. The resulting graph is shown below.

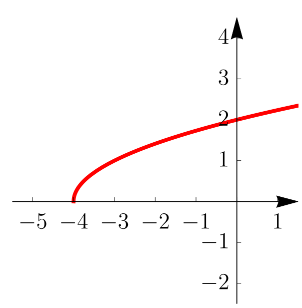


46. a.



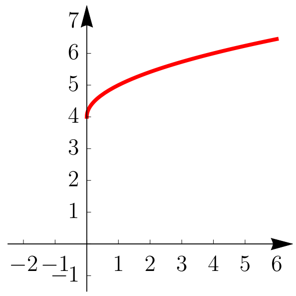
The domain is the set of x-coordinates of points on the graph. From the graph shown above, we see that the domain appears to be $[0, \infty)$. The endpoint of the interval can be verified by noting that $x = 0$ is a valid input value. The range is the set of y-coordinates of points on the graph. Again, using the graph above it appears that the range is $[0, \infty)$. The endpoint can be verified by noting that $y = 0$ when $x = 0$.

b.



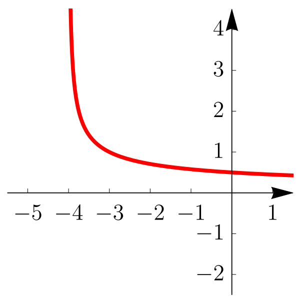
From the graph shown above, we see that the domain appears to be $[-4, \infty)$ and the range appears to be $[0, \infty)$.

c.



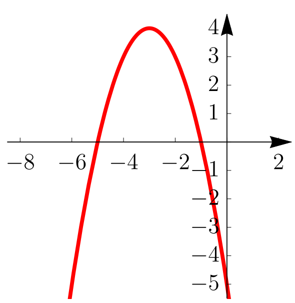
From the graph shown above, we see that the domain appears to be $[0, \infty)$ and the range appears to be $[4, \infty)$.

d.

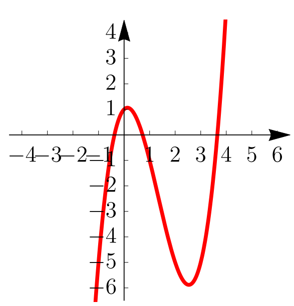


From the graph shown above, we see that the domain appears to be $(-4, \infty)$. The endpoint of the interval is not in the domain since $x = -4$ is not a valid input value. Again, using the graph above it appears that the range is $(0, \infty)$. The endpoint is not in the range since 0 is not an output value of the function.

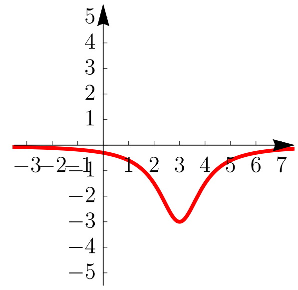
48. The graph, shown below, changes direction at $(-3, 4)$, and so this is a turning point. As we move from left to right in the interval $(-\infty, -3)$, the graph is rising, and so f is increasing on that interval. However, the graph is falling as we move from left to right in the interval $(-3, \infty)$, so f is decreasing on that interval.



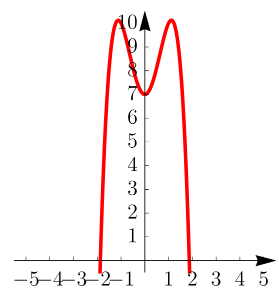
50. The graph, shown below, changes direction at approximately $(0.1, 1.061)$ and $(2.5, -5.875)$, and so these are turning points. As we move from left to right in the interval $(-\infty, 0.1)$, the graph is rising, and so f is increasing on that interval. The graph is falling as we move from left to right in the interval $(0.1, 2.5)$, so f is decreasing on that interval. Finally, the graph is again rising as we move from left to right in the interval $(2.5, \infty)$, so f is increasing on that interval.



52. The graph, shown below, changes direction at $(3, -3)$, and so that is a turning point. As we move from left to right in the interval $(-\infty, 3)$, the graph is falling, and so f is decreasing on that interval. However, the graph is rising as we move from left to right in the interval $(3, \infty)$, so f is increasing on that interval.



54. The graph, shown below, changes direction at approximately $(-1.1, 10.1)$, $(0, 7)$, and $(1.1, 10.1)$, and so these are turning points. As we move from left to right in the interval $(-\infty, -1.1)$, the graph is rising, and so f is increasing on that interval. The graph is falling as we move from left to right in the interval $(-1.1, 0)$, so f is decreasing on that interval. The graph is rising as we move from left to right in the interval $(0, 1.1)$, so f is increasing on that interval. Finally, the graph is again falling as we move from left to right in the interval $(1.1, \infty)$, so f is decreasing on that interval.



56. The first four terms of the sequence $u_n = 4 - n$ are:
$u_1 = 4 - 1 = 3$
$u_2 = 4 - 2 = 2$
$u_3 = 4 - 3 = 1$
$u_4 = 4 - 4 = 0$

58. The first four terms of the sequence $c_n = -\dfrac{1}{2^n}$ are:
$c_1 = -\dfrac{1}{2^1} = -\dfrac{1}{2}$
$c_2 = -\dfrac{1}{2^2} = -\dfrac{1}{4}$
$c_3 = -\dfrac{1}{2^3} = -\dfrac{1}{8}$
$c_4 = -\dfrac{1}{2^4} = -\dfrac{1}{16}$

60. The first four terms of the sequence $b_n = \dfrac{n-1}{n+1}$ are:
$b_1 = \dfrac{1-1}{1+1} = \dfrac{0}{2} = 0$
$b_2 = \dfrac{2-1}{2+1} = \dfrac{1}{3}$
$b_3 = \dfrac{3-1}{3+1} = \dfrac{2}{4} = \dfrac{1}{2}$
$b_4 = \dfrac{4-1}{4+1} = \dfrac{3}{5}$

62. The first four terms of the sequence $x_n = (-1)^n n^2$ are:
$x_1 = (-1)^1 1^2 = (-1)(1) = -1$
$x_2 = (-1)^2 2^2 = (1)(4) = 4$
$x_3 = (-1)^3 3^2 = (-1)(9) = -9$
$x_4 = (-1)^4 4^2 = (1)(16) = 16$

64. The first four terms of the sequence $r_n = \dfrac{(-1)^{n+1}}{3^n}$ are:
$r_1 = \dfrac{(-1)^2}{3^1} = \dfrac{1}{3}$
$r_2 = \dfrac{(-1)^3}{3^2} = -\dfrac{1}{9}$
$r_3 = \dfrac{(-1)^4}{3^3} = \dfrac{1}{27}$
$r_4 = \dfrac{(-1)^5}{3^4} = -\dfrac{1}{81}$

66. The first four terms of the sequence $r_n = \dfrac{n^2}{2^n}$ are:
$r_1 = \dfrac{1^2}{2^1} = \dfrac{1}{2}$
$r_2 = \dfrac{2^2}{2^2} = \dfrac{4}{4} = 1$
$r_3 = \dfrac{3^2}{2^3} = \dfrac{9}{8}$
$r_4 = \dfrac{4^2}{2^4} = \dfrac{16}{16} = 1$

68. The first four terms of the sequence $a_n = 1 \cdot 2 \cdot \cdots \cdot n$ are:
$a_1 = 1$
$a_2 = 1 \cdot 2 = 2$
$a_3 = 1 \cdot 2 \cdot 3 = 6$
$a_4 = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

70. We want to define a_n so that

$$a_1 = 3,\ a_2 = 9,\ a_3 = 27,\ a_4 = 81$$

Since each term is three times the previous term and the first term is 3, we have $a_n = 3^n$.

72. We want to define a_n so that

$$a_1 = 3,\ a_2 = 7,\ a_3 = 11,\ a_4 = 15$$

Notice that each term is four more than the preceding term, which implies that the term should be something like $b_n = 4n$. The sequence b_n begins $4, 8, 12, 16$ and we can see that the sequence we desire is one less than each term in the sequence b_n, so we have that $a_n = b_n - 1 = 4n - 1$.

74. We want to define a_n so that

$$a_1 = 2 \cdot 3,\ a_2 = 3 \cdot 4,\ a_3 = 4 \cdot 5,\ a_4 = 5 \cdot 6$$

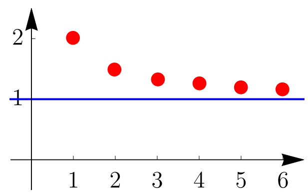
Since each term is one more than the index multiplied by two more than the index, we have $a_n = (n+1)(n+2)$.

76. We want to define a_n so that

$$a_1 = 1 + \frac{1}{2},\ a_2 = 1 - \frac{1}{4},\ a_3 = 1 + \frac{1}{8},\ a_4 = 1 - \frac{1}{16}$$

Begin by noting that each each term begins with a 1 to which a fraction is added or subtracted. This fraction is $\frac{1}{2}$ multipled by itself as many times as the index states, so the fraction for the nth term will be $\frac{1}{2^n}$. Note that the fraction is subtracted when the index is even while it is added if the index is odd. This may be included by mutliplying the fraction for the nth term by $(-1)^{n+1}$, as this number is 1 when n is odd and -1 when n is even. Combining the above observations, we have $a_n = 1 + (-1)^{n+1}\frac{1}{2^n}$.

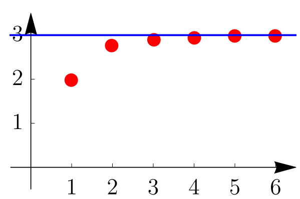
78. Using the values of n as x-coordinates and the corresponding a_n-values as the y-coordinates, we obtain the graph shown below.



a. As we can see, the points seem to be falling. Therefore, the terms of the sequence are decreasing.

b. As n increases, the corresponding points appear to be getting closer and closer to the line $y = 1$. Thus, we conclude that the terms are approaching 1.

80. Using the values of n as x-coordinates and the corresponding a_n-values as the y-coordinates, we obtain the graph shown below.



a. As we can see, the points seem to be rising. Therefore, the terms of the sequence are increasing.

b. As n increases, the corresponding points appear to be getting closer and closer to the line $y = 3$ (the x-axis). Thus, we conclude that the terms are approaching 3.

82. Starting with $a_1 = 1$ and applying the recursive definition $a_n = a_{n-1} + 6$ for $n = 2, 3, 4, 5$, we obtain

$$\begin{aligned} a_1 &= 1 \\ a_2 &= a_1 + 6 = 1 + 6 = 7 \\ a_3 &= a_2 + 6 = -7 + 6 = 13 \\ a_4 &= a_3 + 6 = 13 + 6 = 19 \\ a_5 &= a_4 + 6 = 19 + 6 = 25 \end{aligned}$$

84. Starting with $a_0 = 12$ and applying the recursive definition $a_n = \frac{3}{2}a_{n-1}$ for $n = 1, 2, 3, 4$, we obtain

$$\begin{aligned} a_0 &= 12 \\ a_1 &= \frac{3}{2}a_0 = \frac{3}{2}(12) = 18 \\ a_2 &= \frac{3}{2}a_1 = \frac{3}{2}(18) = 27 \\ a_3 &= \frac{3}{2}a_2 = \frac{3}{2}(27) = \frac{81}{2} \\ a_4 &= \frac{3}{2}a_3 = \frac{3}{2} \cdot \frac{81}{2} = \frac{243}{4} \end{aligned}$$

86. Starting with $a_1 = 1$ and $a_2 = 2$ and applying the recursive definition $a_n = \dfrac{a_{n-1}}{a_{n-2}}$ for $n = 3, 4, 5$, we obtain

$$\begin{aligned} a_1 &= 1 \\ a_2 &= 2 \\ a_3 &= \frac{a_2}{a_1} = \frac{2}{1} = 2 \\ a_4 &= \frac{a_3}{a_2} = \frac{2}{2} = 1 \\ a_5 &= \frac{a_4}{a_3} = \frac{1}{2} \end{aligned}$$

88. The radius can expressed in terms of the diameter as $r = \frac{d}{2}$. Substituting $\frac{d}{2}$ for r in the area formula gives us

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{d}{2}\right)^2 \\ &= \pi \frac{d^2}{4} \\ &= \frac{1}{4}\pi d^2 \\ &= \frac{\pi d^2}{4} \end{aligned}$$

90. Consider the triangle formed from two radii of the sphere extending from the center of the sphere to two adjacent corners of the inscribed cube, together with the side of the cube joining these two corners. The resulting triangle is a right isosceles triangle with legs each of length r and hypotenuse of length s. Thus the radius r and the side s may be related by the Pythagorean Theorem: $r^2 + r^2 = s^2$. The side can be expressed in terms of the radius as $s = \sqrt{2r^2} = r\sqrt{2}$. Substituting $r\sqrt{2}$ for s in the volume formula for a cube gives us

$$\begin{aligned} V &= s^3 \\ &= (r\sqrt{2})^3 \\ &= r^3(\sqrt{2})^3 \\ &= r^3(2\sqrt{2}) \\ &= 2r^3\sqrt{2} \end{aligned}$$

92. a. The mill worker's observation is that $D = L - 2$.

b. The relation $D = L - 2$ can be solved for L to find that $L = D + 2$. This relation can be substituted into the volume formula to find

$$\begin{aligned} V &= (0.79D^2 - 2D - 4)L \\ &= (0.79D^2 - 2D - 4)(D + 2) \\ &= (0.79D^3 - 2D^2 - 4D) + 2(0.79D^2 - 2D - 4) \\ &= 0.79D^3 - 0.42D^2 - 8D - 8 \\ &= f(D) \end{aligned}$$

c. The relation $D = L - 2$ can be substituted into the volume formula to find

$$\begin{aligned} V &= (0.79D^2 - 2D - 4)L \\ &= (0.79(L-2)^2 - 2(L-2) - 4)L \\ &= (0.79(L^2 - 4L + 4) - 2L + 4 - 4)L \\ &= (0.79L^2 - 5.16L + 3.16)L \\ &= 0.79L^3 - 5.16L^2 + 3.16L \\ &= g(L) \end{aligned}$$

94. Since $h = 1$, the formula for the volume of the spill in cubic centimeters is $V = \pi r^2 h = \pi r^2$. But at time t, $V(t) = 10000000t$, so $10000000t = \pi r^2$. That relation may be solved for r to find the radius r in centimeters as a function of time t in minutes: $r = \sqrt{\dfrac{10000000t}{\pi}}$ cm. Two hours after the spill $t = 120$, so

$$r = \sqrt{\frac{1200000000}{\pi}} \approx 19544.1 \text{ cm}$$

96. a. The wages lost are $500 \times 8 = 4000$ dollars, so the total of lost wages and medical bills is $M + 4000$. By the rule of thumb, the value of pain and suffering will be $3(M + 4000)$.
   b. The total amount $S(M)$ of the settlement will include damage, lost wages, medical bills, and pain and suffering. Thus

$$\begin{aligned} S(M) &= 20000 + M + 4000 + 3(M + 4000) \\ &= 4M + 36000 \end{aligned}$$

   c. Since the medical bill amount $M > 0$, the domain of S is $[0, \infty)$. Since S is a linear function that is increasing, its lowest value will be $S(0) = 36000$. Thus the range of S is $[36000, \infty)$.
   d. $S(1000) = 4(1000) + 36000 = 40000$, so the total settlement will be approximately $40,000.
   e. The equation $S(M) = 4M + 36000 = 100000$ may be solved to find that $M = 16000$. Thus $16,000 was spent on medical bills.
   f. The slope of the line $S(M) = 4M + 36000$ is 4, which means that for each extra dollar spent on medical expenses, $4 was added to the settlement. Thus $4 profit would be added for each dollar spent on unnecessary medical expenses.

98. a. The total number of people in year $n + 1$ will be the number of people present in year n plus k new people for every 1000 people present at time n. If P_n is the number of people at time n, the number of new people will be $\dfrac{k}{1000}P_n$, so the total number of people at time $n + 1$ will be

$$P_{n+1} = P_n + \frac{k}{1000}P_n = \left(1 + \frac{k}{1000}\right)P_n$$

   b. Since $P_{n+1} = \left(1 + \dfrac{k}{1000}\right)P_n$, the sequence P_n is geometric with $r = 1 + \dfrac{k}{1000}$.
   c. Note that $P_1 = \left(1 + \dfrac{k}{1000}\right)P_0$, $P_2 = \left(1 + \dfrac{k}{1000}\right)P_1 = \left(1 + \dfrac{k}{1000}\right)^2 P_0$, $P_3 = \left(1 + \dfrac{k}{1000}\right)P_2 = \left(1 + \dfrac{k}{1000}\right)^3 P_0$ and so on. Thus $P_n = \left(1 + \dfrac{k}{1000}\right)^n P_0$.
   d. Since $k = 58$ and $P_0 = 478434$ in 2000, the population in year $2000 + n$ will be

$$P_n = \left(1 + \frac{58}{1000}\right)^n (478434) = 478434(1.058)^n$$

   Thus in 2050, $n = 50$ and population would be approximately $478434(1.058)^{50} = 8018727$.

1.2 Library of Functions

6. The domain of a rational function will not include points where the denominator is 0, so the function $f(x) = \dfrac{1}{x-2}$ is an example of a function for which $x = 2$ is not in the domain.

8. The power function $f(x) = x^\pi$ does not use rational powers of the variable, and so is not an algebraic function.

10. This is an algebraic function as it the composition of a polynomial function (x^2+1) with a power function with rational exponent $\frac{1}{2}$. It is not a polynomial and hence not linear, nor is it a rational function. No algebraic function is transcendental. Thus, f is algebraic.

12. This is an algebraic function as it the sum of a polynomial function $(x^2 + 2x)$ with a power function with rational exponent $\frac{1}{2}$. It is not a polynomial and hence not linear, nor is it a rational function. No algebraic function is transcendental. Thus, f is algebraic.

14. This is a linear function, thus it is also a polynomial and a rational function and an algebraic function. No algebraic function is transcendental. Thus, f is linear, polynomial, rational, and algebraic.

16. This is not an algebraic function as it not an arithmetic combination or composition of polynomials, rational functions, or power functions with rational exponents. Thus it is not a linear function, a polynomial, or a rational function. Since it is not an algebraic function, the function is transcendental. Thus, f is transcendental.

18. Since the line has x-intercept 4 and y-intercept 2, the line passes through $(4,0)$ and $(0,2)$. First, we find the slope of the line passing through those points.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(2-0)}{(0-4)} \\ &= \frac{2}{-4} \\ &= -\frac{1}{2} \end{aligned}$$

To express the equation in slope-intercept form, we note that the slope of the line is $m = -\frac{1}{2}$ and the y-intercept is $b = 2$, so

$$\begin{aligned} y &= mx + b \\ y &= -\frac{1}{2}x + 2 \end{aligned}$$

20. For our line to be parallel to the line containing $(0,3)$ and $(4,0)$, the slope must be equal to the slope of the line containing $(0,3)$ and $(4,0)$. The slope of that line is

$$\frac{0-3}{4-0} = -\frac{3}{4},$$

so the slope of our line is $m = \frac{1}{3}$. Using this slope and the given point $(-2,5)$, we can find an equation in point-slope form, and from there, rewrite the equation so that it is in slope-intercept form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= -\frac{3}{4}(x - (-2)) \\ y - 5 &= -\frac{3}{4}x - \frac{3}{2} \\ y &= -\frac{3}{4}x + \frac{7}{2} \end{aligned}$$

22. We start by finding the point of intersection of $2x + y = 10$ and $x - 2y = 5$. We solve $2x + y = 10$ for y to find that $y = 10 - 2x$. Substituting this value for x into $x - 2y = 5$ gives $x - 2(10 - 2x) = 5$, or $x - 20 + 4x = 5$. Thus $5x = 25$, or $x = 5$. The y-coordinate of the point of intersection is $y = 10 - 2(5) = 0$. Thus the point of intersection of $2x + y = 10$ and $x - 2y = 5$ is $(5, 0)$. The line must now pass through $(0, 0)$ and $(5, 0)$ so the slope of that line is

$$m = \frac{0-0}{5-0} = \frac{0}{5} = 0$$

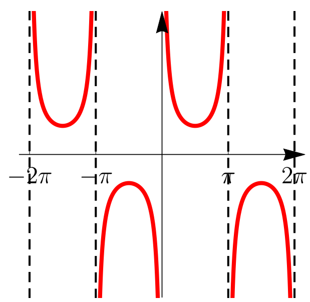
The given point $(0, 0)$ implies that the y-intercept of the line is $b = 0$, we can find an equation in in slope-intercept form.

$$\begin{aligned} y &= mx + b \\ y &= 0x + 0 \\ y &= 0 \end{aligned}$$

24. Since $\csc x = 1/\sin x$, the domain consists of all real numbers except those for which $\sin x = 0$. Since the sine of an angle corresponds to the y-coordinate of a point on the unit circle, and since y-coordinates of points are 0 along the x-axis, we know that $\sin x = 0$ for $x = 0$, $x = \pi$, and any other point of the form $x = n\pi$ where n is an integer. Therefore, the domain of $f(x) = \csc x$ is

$$\{x \mid x \neq n\pi \text{ for any integer } n\}$$

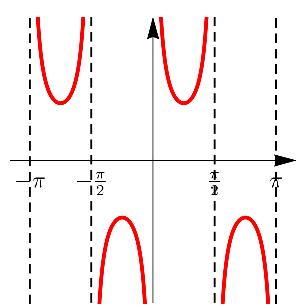
The graph of $f(x) = \csc x$, shown below, indicates a period of 2π.



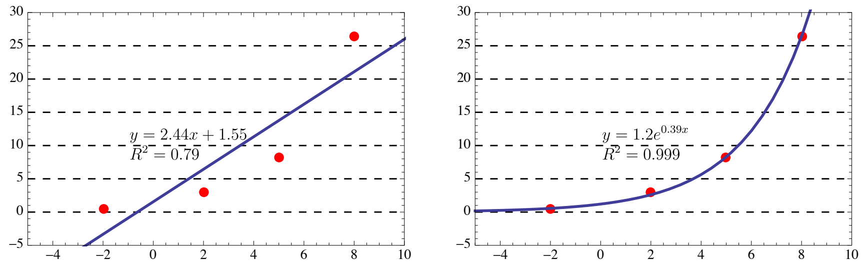
26. Since $\tan x = \sin x/\cos x$ and $\cot x = \cos x/\sin x$, the domain consists of all real numbers except those for which $\cos x = 0$ and $\sin x = 0$. We know that that $\cos x = 0$ for all points of the form $x = \pm\frac{k\pi}{2}$ where k is an odd number and that $\sin x = 0$ for all points of the form $x = n\pi$ where n is an integer. If we combine both conditions, then the domain of $f(x) = \tan x + \cot x$ is

$$\{x \mid x \neq \tfrac{n\pi}{2} \text{ for any integer } n\}$$

The graph of $f(x) = \tan x + \cot x$, shown below, indicates a period of π.

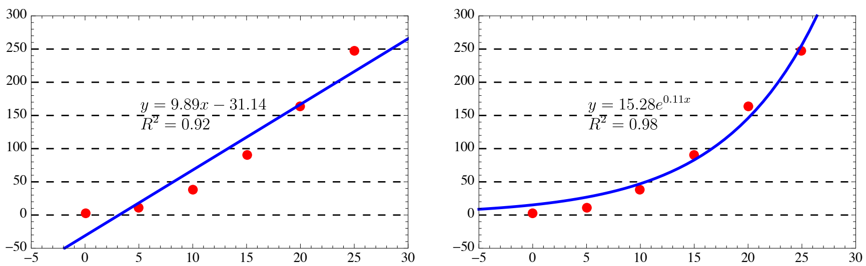


28. a. Using a graphing utility, a CAS, or a spreadsheet, we obtain the following.



b. Since the R^2 value is larger for the exponential model, that model provides a better fit to the data.

30. a. Using a graphing utility, a CAS, or a spreadsheet, we obtain the following.



b. Since the R^2 value is larger for the exponential model, that model provides a better fit to the data.

32. a. The slope of the line will be $m = \dfrac{100-0}{212-32} = \dfrac{5}{9}$ and the line passes through the point $(32, 0)$, so $C - 0 = \dfrac{5}{9}(F-32)$, or $C = \dfrac{5}{9}(F-32) = \dfrac{5}{9}F - \dfrac{160}{9}$.

b. $C = \dfrac{5}{9}(72-32) = \dfrac{200}{9} = 22.22°\text{C}$

c. $C = \dfrac{5}{9}(98.6-32) = \dfrac{333}{9} = 37°\text{C}$

d. The slope is the amount by which the Celsius temperature increases for each increase of 1°F.

34. a. The slope of the line will be $m = 0.1$ and the line passes through the point $(10000, 2000)$, so $P - 2000 = 0.1(S - 10000)$, or $P = 2000 + 0.1(S - 10000) = 1000 + 0.1S$.

b. $P(25000) = 1000 + 0.1(25000) = \$3500$

36. a. The slope of the line will be $m = \dfrac{448-232}{1000-500} = 0.432$ and the line passes through the point $(500, 232)$, so $P - 232 = 0.432(d - 500)$, or $P = 232 + 0.432(d - 500) = .432d + 16$.

b. The slope of the line is the amount that the pressure increases for each additional foot of depth. The y-intercept is the pressure at ground level.

c. $P = .432(36000) + 16 = 15,568$ pounds per square inch.

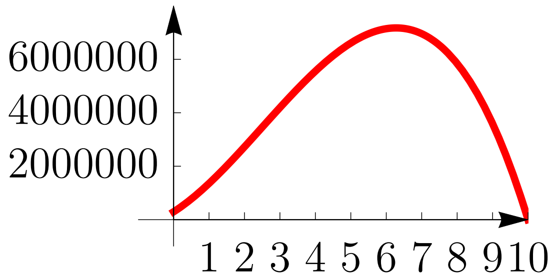
38. a. Since $f(t) \le 0$ for $t \ge 10$, the price per unit model will not be valid in years 2009 and beyond.

b. The revenue will be $h(t) = f(t)g(t) = (2350 - 235t)(146t^2 + 370t + 125)$. This function is a polynomial of degree 3.

c. Since h is a polynomial function, it cannot approach a fixed value as t increases.

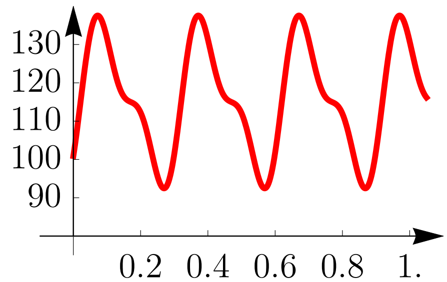
d. The function h gives a plausible result for $0 \le t \le 10$, or the years 1999-2009.

e. Using the graph of h, we see that the total revenue will peak in 2005.



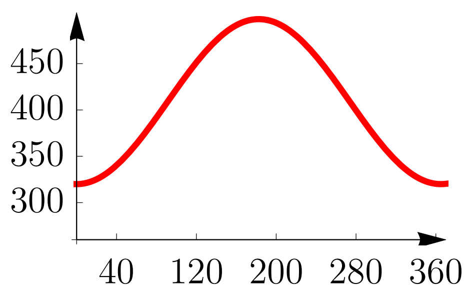
40. a. Since $f(0) = 100.9$ and $f(\frac{\pi}{21}) = 116.5$, we can surmise that the dog's blood pressure is rising immediately after $t = 0$.

   b. Using the graph of f, the highest blood pressure is near 138, and the lowest is near 92.



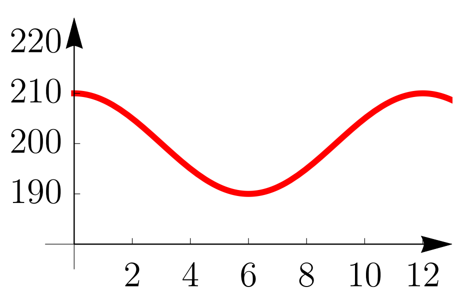
42. a. Since $f(31) = 332$, the sunset time for $t = 31$ is 5:32 P.M. Since $f(150) = 484$, the sunset time for $t = 31$ is 8:04 P.M. Since $f(310) = 357$, the sunset time for $t = 310$ is 5:57 P.M.

   b. The least value of f is $-89 + 409 = 320$, so the earliest sunset time is 5:20 P.M.. The greatest value of f is $89 + 409 = 498$, so the latest sunset time is 8:18 P.M. The least value of f occurs at $t = 0$, so the earliest sunset occurs on January 1. The greatest value of f occurs at $t = 182.5$, so the latest sunset occurs on July 1.



44. a. The greatest value of w is $10(1) + 200 = 210$, which occurs when $\frac{\pi}{6}t = 0$, or $t = 0$. The date at $t = 0$ is January 1.

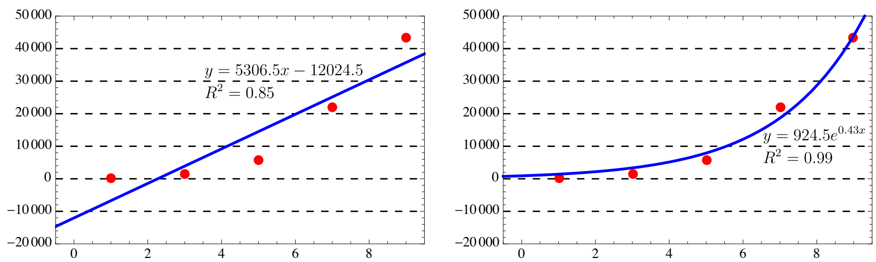
   b. The least value of w is $10(-1) + 200 = 190$, which occurs when $\frac{\pi}{6}t = \pi$, or $t = 6$. The date at $t = 6$ is July 1.



46. In 2000, $t = 29$ and Moore's law gives an estimate of $N = 2300 \cdot 2^{29/2}$ which is approximately 53 million. This is about 25% higher than the number of transistors on a Pentium 4 processor. In 2005, $t = 34$ and Moore's law gives an estimate of approximately 3 hundred million.

48. a. For ease in calculation let $x = 0$ in 1990. The least-squares best fit line is $5306.5x - 12024.5$. The correlation coefficient for this model is $R^2 = 0.85$.

    b. For ease in calculation let $x = 0$ in 1990. The least-squares best exponential fit is $924.5e^{0.43x}$. The correlation coefficient for this model is $R^2 = 0.99$. Thus the exponential model fits the data better. In 2002, $x = 12$ and this model gives an estimate of $159,356$ internet hosts.



1.3 Implicit Functions and Conic Sections

8. a. The graph passes the vertical line test, so we know that y can be defined as a function of x.

   b. Since y is a function of x, it must also be a function of x locally at each point on the graph.

10. a. This is not a function because it does not pass the vertical line test. For example, the line $x = 1$ crosses the graph twice.

    b. At the point $(-4, -1)$, zooming in on the graph will always show a graph that violates the vertical line test. At the point $(4, -1)$, zooming in on the graph will always show an X-shaped graph that violates the vertical line test.

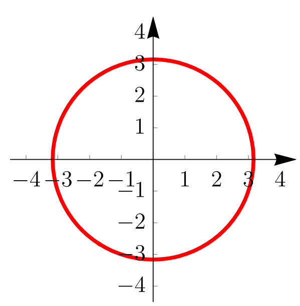
12. The standard form of a circle centered at (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. We are given the equation

$$x^2 + y^2 = 10$$

which is equivalent to

$$(x - 0)^2 + (y - 0)^2 = (\sqrt{10})^2$$

Therefore, this is a circle with center $(0, 0)$ and radius $\sqrt{10}$. The graph is shown below.



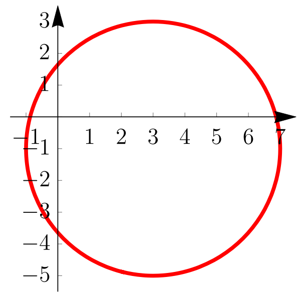
14. The standard form of a circle centered at (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. We are given the equation

$$(x - 3)^2 + (y + 1)^2 = 16$$

which is equivalent to

$$(x - 3)^2 + (y - (-1))^2 = 4^2$$

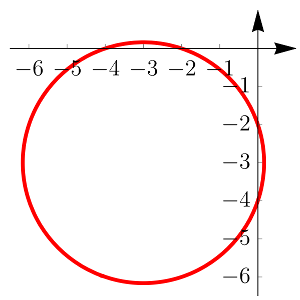
Therefore, this is a circle with center $(3, -1)$ and radius 4. The graph is shown below.



16. By completing the square, we can find the standard equation of the circle.

$$
\begin{aligned}
x^2 + y^2 + 6x + 6y + 8 &= 0 \\
(x^2 + 6x) + (y^2 + 6y) &= -8 \\
(x^2 + 6x + 9) + (y^2 + 6y + 9) &= 10 \\
(x + 3)^2 + (y + 3)^2 &= (\sqrt{10})^2
\end{aligned}
$$

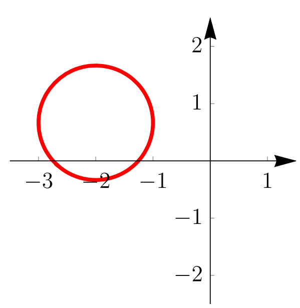
This is the equation of a circle centered at $(-3, -3)$ with a radius of $\sqrt{10}$. The graph is shown below.



18. By dividing the equation by the coefficient of x^2 then completing the square, we can find the standard equation of the circle.

$$
\begin{aligned}
9x^2 + 9y^2 + 36x - 12y + 31 &= 0 \\
x^2 + y^2 + 4x - \frac{4}{3}y + \frac{31}{9} &= 0 \\
(x^2 + 4x) + \left(y^2 - \frac{4}{3}y\right) &= -\frac{31}{9} \\
(x^2 + 4x + 4) + \left(y^2 - \frac{4}{3}y + \frac{4}{9}\right) &= -\frac{31}{9} + 4 + \frac{4}{9} = 1 \\
(x + 2)^2 + \left(y - \frac{2}{3}\right)^2 &= 1^2
\end{aligned}
$$

This is the equation of a circle centered at $(-2, \frac{2}{3})$ with a radius of 1. The graph is shown below.



20. The equation of a circle centered at $(4, 0)$ with radius 2 is

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\(x-4)^2+(y-0)^2&=4\\(x-4)^2+y^2&=4\end{aligned}$$

22. We begin by finding the radius of the circle, which is the distance between the center and a point on the circle. Using the distance formula with the points $(-3, -6)$ and $(-3, 0)$, we have

$$\begin{aligned}r&=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\&=\sqrt{(-3-(-3))^2+(0-(-6))^2}\\&=\sqrt{(0)^2+(6)^2}\\&=\sqrt{36}\\&=6\end{aligned}$$

An equation of the circle with center $(-3, -6)$ and radius 6 is

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\(x+3)^2+(y+6)^2&=36\end{aligned}$$

24. The center of the circle is the point midway between the endpoints $(-3, 6)$ and $(5, -6)$. Using the midpoint formula, the center is thus the point $(\overline{x}, \overline{y})$ where

$$\overline{x}=\frac{x_1+x_2}{2}=\frac{-3+5}{2}=\frac{2}{2}=1$$

and

$$\overline{y}=\frac{y_1+y_2}{2}=\frac{6+(-6)}{2}=\frac{0}{2}=0$$

Since the radius is the distance between the center and any point on the circle, we apply the distance formula to the points $(-3, 6)$ and $(1, 0)$

$$\begin{aligned}r&=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\&=\sqrt{(1-(-3))^2+(0-6)^2}\\&=\sqrt{(4)^2+(-6)^2}\\&=\sqrt{52}\\&=2\sqrt{13}\end{aligned}$$

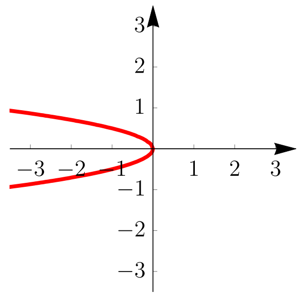
Thus, the standard equation of the circle with center $(1, 0)$ and radius $2\sqrt{13}$ is

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\(x-1)^2+(y-0)^2&=\left(2\sqrt{13}\right)^2\\(x-1)^2+y^2&=52\end{aligned}$$

26. The standard form for the equation of a parabola with vertex (h, k) and a horizontal axis of symmetry is $x-h=a(y-k)^2$. We are given the equation

$$x=-4y^2$$

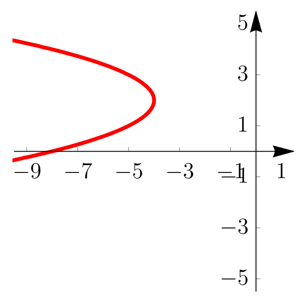
In this case, both h and k are 0, and so the vertex is the point $(0, 0)$ and the axis of symmetry is the horizontal line $y = 0$. The graph is shown below.



28. The standard equation of a parabola with a horizontal axis of symmetry is $x - h = a(y - k)^2$. We have

$$x + 4 = -1(y - 2)^2$$

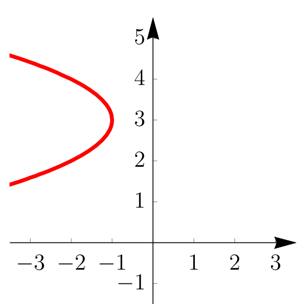
Thus, the vertex is the point $(-4, 2)$, and the axis of symmetry is the horizontal line $y = 2$. The graph of the parabola is shown below.



30. We begin by completing the square to write the equation in standard form, $x - h = a(y - k)^2$.

$$\begin{aligned} y^2 + x - 6y + 10 &= 0 \\ x + 10 &= -y^2 + 6y \\ x + 10 &= -(y^2 - 6y + \boxed{}) \\ x + 10 - 9 &= -(y^2 - 6y + \boxed{9}) \\ x + 1 &= -(y - 3)^2 \end{aligned}$$

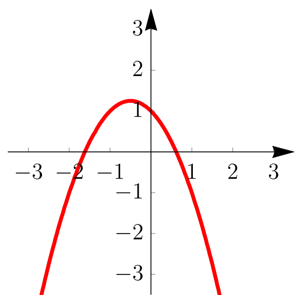
Thus, the vertex is the point $(-1, 3)$, and the axis of symmetry is the horizontal line $y = 3$. The graph is shown below.



32. By rewriting the equation of the parabola in standard form, we can find the coordinates of the vertex.

$$\begin{aligned} y &= -x^2 - x + 1 \\ y - 1 &= -x^2 - x \\ y - 1 &= -1(x^2 + x + \boxed{}) \\ y - 1 - \frac{1}{4} &= -1(x^2 + x + \boxed{\tfrac{1}{4}}) \\ y - \frac{5}{4} &= -1\left(x + \frac{1}{2}\right)^2 \end{aligned}$$

From the standard form, we can see that the vertex of the parabola is $\left(-\frac{1}{2}, \frac{5}{4}\right)$, and its axis of symmetry is vertical. Using a graphing utility (or plotting a few points and connecting with a smooth curve), we obtain the graph shown below.



34. The standard equation of a parabola with a horizontal axis of symmetry is $x - h = a(y - k)^2$. Since the vertex is $(0, 0)$, both h and k are 0 and we have

$$\begin{aligned} x - 0 &= a(y - 0)^2 \\ x &= ay^2 \end{aligned}$$

Using the fact that $(4, 3)$ is on the parabola gives us

$$\begin{aligned} 4 &= a(3)^2 \\ \frac{4}{9} &= a \end{aligned}$$

So the equation of the parabola is $x = \frac{4}{9}y^2$.

36. The standard equation of a parabola with a vertical axis of symmetry is $y - k = a(x - h)^2$. Since the vertex is $(-2, -4)$, $h = -2$ and $k = -4$ and we have

$$y + 4 = a(x + 2)^2$$

Using the fact that $(0, -6)$ is on the parabola gives us

$$\begin{aligned} -2 &= a(2)^2 \\ -\frac{1}{2} &= a \end{aligned}$$

So the equation of the parabola is $y + 4 = -\frac{1}{2}(x + 2)^2$.

38. The standard equation for a parabola with horizontal axis of symmetry is $x - h = a(y-k)^2$. Since the points $(0,0)$, $(-2,-10)$, and $(2,2)$ lie on the parabola, their coordinates must satisfy the equation $x = ay^2 + by + c$. For example, setting $x = 0$ and $y = 0$ we have

$$\begin{aligned} 0 &= a(0)^2 + b(0) + c \\ 0 &= c \end{aligned}$$

Now using $c = 0$ and point $(-2,-10)$ we obtain

$$\begin{aligned} x &= ay^2 + by \\ -2 &= a(-10)^2 + b(-10) \\ -2 &= 100a - 10b \qquad (1.1) \end{aligned}$$

Similarly, from the point $(2,2)$ we have

$$\begin{aligned} x &= ay^2 + by \\ 2 &= a(2)^2 + b(2) \\ 2 &= 4a + 2b \qquad (1.2) \end{aligned}$$

We now solve the system of equations given by Equations 1.1 and 1.2. Solving Equation 1.2 for b, we obtain

$$b = -2a + 1 \qquad (1.3)$$

Substituting $-2a + 1$ for b in Equation 1.1, we have

$$\begin{aligned} 100a - 10b &= -2 \\ 100a - 10(-2a+1) &= -2 \\ 120a - 10 &= -2 \\ 120a &= 8 \\ a &= \frac{1}{15} \end{aligned}$$

Finally, substituting $a = \frac{1}{15}$ into Equation 1.3, we obtain $b = \frac{13}{15}$. Thus, the parabola has equation $x = \frac{1}{15}y^2 + \frac{13}{15}y$. To put this in standard form, we complete the square on the right-hand side as follows.

$$\begin{aligned} x &= \frac{1}{15}y^2 + \frac{13}{15}y \\ x &= \frac{1}{15}\left(y^2 + 13y\right) \\ x + \frac{169}{60} &= \frac{1}{15}\left(y^2 + 13y + \frac{169}{4}\right) \\ x + \frac{169}{60} &= \frac{1}{15}\left(y + \frac{13}{2}\right)^2 \end{aligned}$$

40. The standard equation of an ellipse centered at $(0,0)$ is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

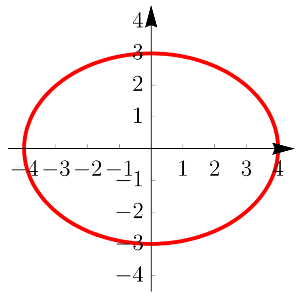
We are given the equation

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

which is equivalent to

$$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$$

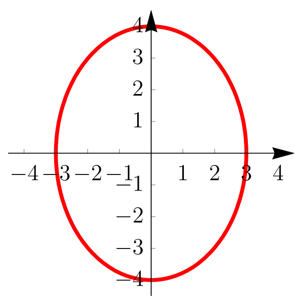
Thus, $a = 4$ and $b = 3$. Since $a > b$, the major axis is horizontal and connects the vertices $(4,0)$ and $(-4,0)$. The minor axis connects $(0,-3)$ and $(0,3)$. The graph is shown below.



42. First, we write the equation in standard form by dividing both sides by 144.

$$16x^2 + 9y^2 = 144$$
$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

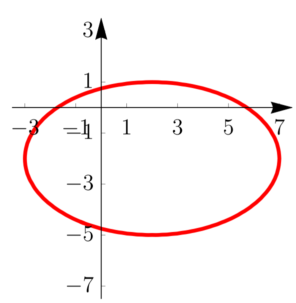
Thus, $a = 3$ and $b = 4$. Since $a < b$, the graph has a vertical orientation. The center of the ellipse is at $(0, 0)$, and so the major axis is vertical with vertices $(0, 4)$ and $(0, -4)$. The minor axis connects $(-3, 0)$ and $(3, 0)$. The graph is shown below.



44. We have

$$\frac{(x-2)^2}{25} + \frac{(y+2)^2}{9} = 1$$
$$\frac{(x-2)^2}{5^2} + \frac{(y+2)^2}{3^2} = 1$$

The center of the ellipse is $(2, -2)$. We see that $a = 5$, $b = 3$, and $a > b$, and so the major axis is horizontal and connects the vertices $(-3, -2)$ and $(7, -2)$. The minor axis connects $(2, -5)$ and $(2, 1)$. The graph is shown below.

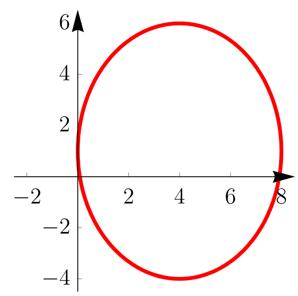


46. By finding the standard equation of an ellipse from the given information, we can identify its center, vertices,

and major and minor axes.

$$\begin{aligned}
25x^2 + 16y^2 - 200x - 32y + 16 &= 0 \\
(25x^2 - 200x) + (16y^2 - 32y) &= -16 \\
25(x^2 - 8x) + 16(y^2 - 2y) &= -16 \\
25(x^2 - 8x + 16) + 16(y^2 - 2y + 1) &= 400 \\
25(x-4)^2 + 16(y-1)^2 &= 400 \\
\frac{(x-4)^2}{16} + \frac{(y-1)^2}{25} &= 1 \\
\frac{(x-4)^2}{4^2} + \frac{(y-1)^2}{5^2} &= 1
\end{aligned}$$

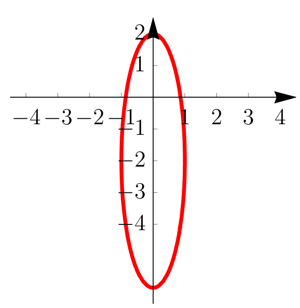
From the standard equation, we see that the center is $(4, 1)$, and that $a = 4$ and $b = 5$. Since $a < b$, the ellipse has a vertical major axis with vertices $(4, -4)$ and $(4, 6)$. The minor axis extends from $(0, 1)$ to $(8, 1)$. The graph is shown below.



48. By finding the standard equation of an ellipse from the given information, we can identify its center, vertices, and major and minor axes.

$$\begin{aligned}
16x^2 + y^2 + 4y - 12 &= 0 \\
16x^2 + (y^2 + 4y) &= 12 \\
16x^2 + (y^2 + 4y + 4) &= 16 \\
16(x-0)^2 + 1(y+2)^2 &= 16 \\
\frac{(x-0)^2}{1} + \frac{(y+2)^2}{16} &= 1 \\
\frac{(x-0)^2}{1^2} + \frac{(y+2)^2}{4^2} &= 1
\end{aligned}$$

From the standard equation, we see that the center is $(0, -2)$, and that $a = 1$ and $b = 4$. Since $a < b$, the ellipse has a vertical major axis with vertices $(0, -6)$ and $(0, 2)$. The minor axis extends from $(-1, -2)$ to $(1, -2)$. The graph is shown below.



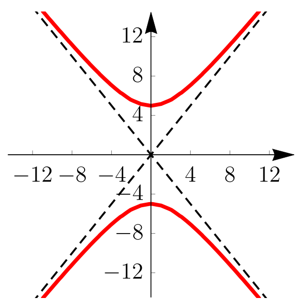
50. The standard form for a hyperbola centered at the origin with a vertical transverse axis is

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

Thus, the graph of $\frac{y^2}{25} - \frac{x^2}{16} = 1$ is a hyperbola centered at the origin with $a = 4$ and $b = 5$. Since the hyperbola has a vertical transverse axis, the vertices are located at $(0, -5)$ and $(0, 5)$.
The asymptotes for a hyperbola centered at the origin are

$$y = \frac{b}{a}x \text{ and } y = -\frac{b}{a}x$$

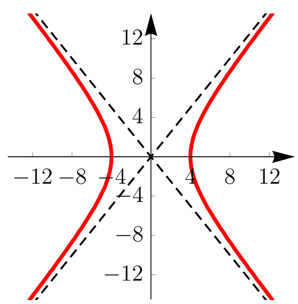
For this hyperbola, that means the asymptotes are $y = \pm\frac{5}{4}x$. The graph is shown below.



52. We begin by rewriting the equation in standard form.

$$\begin{aligned} 25x^2 - 16y^2 &= 400 \\ \frac{x^2}{16} - \frac{y^2}{25} &= 1 \\ \frac{x^2}{4^2} - \frac{y^2}{5^2} &= 1 \end{aligned}$$

The center of the hyperbola is $(0, 0)$. The transverse axis is horizontal with $a = 4$ and $b = 5$. The vertices are located 4 units to the left and right of the center, at $(-4, 0)$ and $(4, 0)$. The asymptotes for a hyperbola centered at the origin are $y = \pm\frac{b}{a}x$. Therefore, for this hyperbola the asymptotes are $y = \pm\frac{5}{4}x$. The graph is shown below.



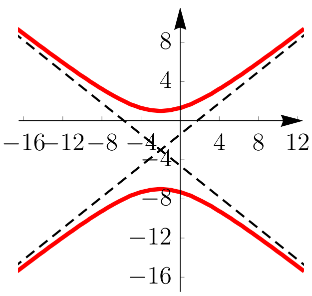
54. The standard equation of a hyperbola with a vertical transverse axis centered at (h, k) is

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Thus, the graph of $\frac{(y+3)^2}{16} - \frac{(x+2)^2}{25} = 1$ is centered at $(-2, -3)$ with $a = 5$ and $b = 4$. It follows that the vertices are 4 units above and below the center at $(-2, 1)$ and $(-2, -7)$. The asymptotes of a hyperbola with a vertical transverse axis centered at (h, k) are

$$y = \pm\frac{b}{a}(x - h) + k$$

so the asymptotes for this hyperbola are $y = \pm\frac{4}{5}(x + 2) - 3$. The graph is shown below.



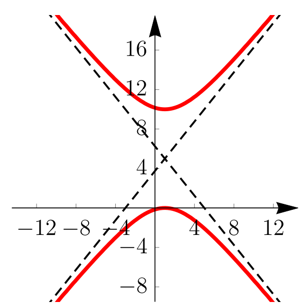
56. We begin by rewriting the equation in standard form by completing the square on the x and y terms.

$$\begin{aligned}
25x^2 - 16y^2 - 50x + 160y + 25 &= 0 \\
25x^2 - 50x - 16y^2 + 160y &= -25 \\
25(x^2 - 2x) - 16(y^2 - 10y) &= -25 \\
25(x^2 - 2x + 1) - 16(y^2 - 10y + 25) &= -400 \\
16(y^2 - 10y + 25) - 25(x^2 - 2x + 1) &= 400 \\
\frac{(y-5)^2}{25} - \frac{(x-1)^2}{16} &= 1
\end{aligned}$$

The hyperbola is centered at $(1, 5)$. The axis is vertical with $a = 4$ and $b = 5$. The vertices are 5 units above and below the center, at $(1, 0)$ and $(1, 10)$. The asymptotes of a hyperbola with a vertical transverse axis centered at (h, k) are

$$y = \pm\frac{b}{a}(x - h) + k$$

so the asymptotes for this hyperbola are $y = \pm\frac{5}{4}(x - 1) + 5$. The graph is shown below.



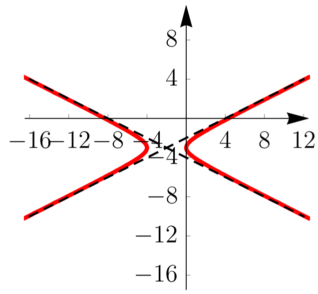
58. We begin by rewriting the equation in standard form by completing the square on the x and y terms.

$$\begin{aligned}
x^2 - 4y^2 + 4x - 24y - 36 &= 0 \\
x^2 + 4x - 4y^2 - 24y &= 36 \\
(x^2 + 4x) - 4(y^2 + 6y) &= 36 \\
(x^2 + 4x + 4) - 4(y^2 + 6y + 9) &= 4 \\
\frac{(x+2)^2}{4} - \frac{(y+3)^2}{1} &= 1
\end{aligned}$$

The hyperbola is centered at $(-2, -3)$. The axis is horizontal with $a = 2$ and $b = 1$. The vertices are 2 units to the left and the right of the center, at $(-4, -3)$ and $(0, -3)$. The asymptotes of a hyperbola with a horizontal transverse axis centered at (h, k) are

$$y = \pm\frac{b}{a}(x - h) + k$$

so the asymptotes for this hyperbola are $y = \pm\frac{1}{2}(x + 2) - 3$. The graph is shown below.



60. We have

$$3x^2 + 2y^2 = 7x - 7y - 5$$
$$3x^2 - 7x + 2y^2 + 7y + 5 = 0$$

The presence of square terms for both x and y, with coefficients that are different but of the same sign, suggests that this is the equation of an ellipse.

62. Since the equation is quadratic in y but linear in x, it is a parabola.

$$4y^2 - 9x + 23y - 9 = 0$$
$$-9x = -4y^2 - 23y + 9$$
$$x = \frac{4}{9}y^2 + \frac{23}{9}y - 1$$

This simplification is sufficient to show that the given conic section is a parabola.

64. We have

$$x^2 - 2y^2 + 5 = 0$$
$$x^2 - 2y^2 = -5$$

The presence of the square terms for both x and y, with coefficients of opposite sign, suggests that this is the equation of a hyperbola.

66. The equation may be rewritten as $3x^2 + 3y^2 = 5y + 2$. The presence of the square terms for x and y with identical coefficients suggests that this is a circle.

68. From the problem we see that $h = 100$. If the car is to land 200 feet from the base of the garage, then $x = 200$ when $y = 0$. Plugging these values into the equation above gives

$$0 = 100 - \frac{16}{v^2}(200)^2$$
$$-100 = -40000\frac{16}{v^2}$$
$$\frac{1}{400} = \frac{16}{v^2}$$
$$v^2 = 6400$$
$$v = 80$$

70. a. From the standard equation of a parabola with vertex $(0, 0)$ and a vertical axis, we have

$$y = ax^2$$

Since the parabola passes through the point $(100, 40)$, substituting $(30, 0)$ in our formula gives us

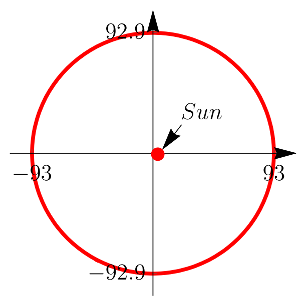
$$40 = a(100)^2$$
$$40 = 10000a$$
$$a = \frac{1}{250}$$

Thus, the equation of the shape of the cable is the parabola $y = \frac{1}{250}x^2$.

b. The lengths of the vertical cables will be the height of the cable $y = \frac{1}{250}x^2$ evaluated at the values $x = -80, -60, -40, -20, 20, 40, 60, 80$. These lengths are $25.6, 14.4, 6.4, 1.6, 1.6, 6.4, 14.4, 25.6$ so the total length required is 96 feet, or 192 feet if cables on both sides of the highway are considered.

72. The center of the ellipse is at $(h, k) = (0, 0)$ with $a = \frac{186000000}{2} = 93000000$ and $b = \frac{185800000}{2} = 92900000$. Thus the equation of the ellipse could be

$$\frac{x^2}{93000000^2} + \frac{y^2}{92900000^2}$$



1.4 Polar Functions

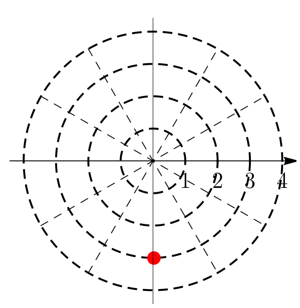
6. A line passing through the origin can be represented by $\theta = k$ for any k, and so one example is $\theta = \frac{\pi}{4}$.

8. Consider the polar curve $r = k \sin\theta$. Multiplying both sides of the equation by r and converting to rectangular coordinates gives:

$$r^2 = kr\sin\theta$$
$$x^2 + y^2 = ky$$
$$x^2 + y^2 - ky = 0$$
$$x^2 + y^2 - ky + \frac{k^2}{4} = \frac{k^2}{4}$$
$$(x-0)^2 + \left(y - \frac{k}{2}\right)^2 = \left(\frac{k}{2}\right)^2$$

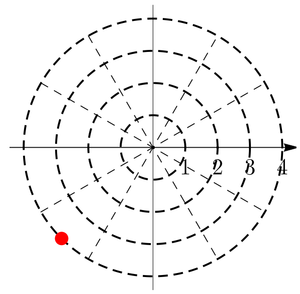
which is the equation of a circle with radius $\frac{k}{2}$ and center $\left(0, \frac{k}{2}\right)$. Thus an example would be $r = 3\sin\theta$.

10. First, we plot the point.



We see that the point is 3 units below the origin on the y-axis, and so the rectangular coordinates are $(0, -3)$.

12. The point with polar coordinates $\left(-4, \frac{\pi}{4}\right)$ is shown below.

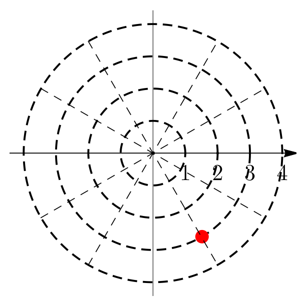


With $r = -4$ and $\theta = \frac{\pi}{4}$, we have

$$\begin{aligned} x &= r\cos\theta \\ &= (-4)\cos\frac{\pi}{4} \\ &= -2\sqrt{2} \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= (-4)\sin\frac{\pi}{4} \\ &= -2\sqrt{2} \end{aligned}$$

Thus, the point has rectangular form $(-2\sqrt{2}, -2\sqrt{2})$.

14. The point with polar coordinates $\left(3, -\frac{\pi}{3}\right)$ is shown below.

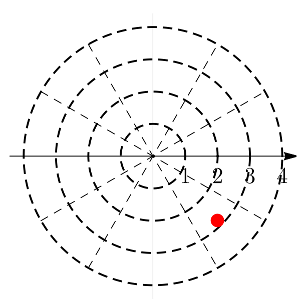


With $r = 3$ and $\theta = -\frac{\pi}{3}$, we have

$$\begin{aligned} x &= r\cos\theta \\ &= (3)\cos\left(-\frac{\pi}{3}\right) \\ &= \frac{3}{2} \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= (3)\sin\left(-\frac{\pi}{3}\right) \\ &= -\frac{3\sqrt{3}}{2} \end{aligned}$$

Thus, the point has rectangular form $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$.

16. The point with polar coordinates $\left(2\sqrt{2}, \frac{7\pi}{4}\right)$ is shown below.

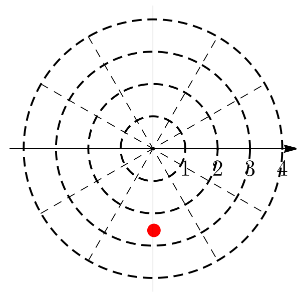


With $r = 2\sqrt{2}$ and $\theta = \frac{7\pi}{4}$, we have

$$x = r\cos\theta = (2\sqrt{2})\cos\frac{7\pi}{4} = 2 \qquad y = r\sin\theta = (2\sqrt{2})\sin\frac{7\pi}{4} = -2$$

Thus, the point has rectangular form $(2, -2)$.

18. The point with polar coordinates $\left(-\frac{5}{2}, -\frac{7\pi}{2}\right)$ is shown below.

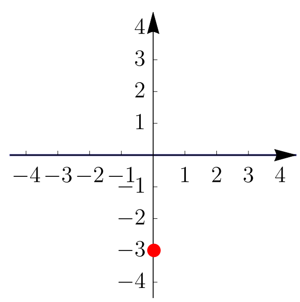


With $r = -\frac{5}{2}$ and $\theta = -\frac{7\pi}{2}$, we have

$$x = r\cos\theta = (-\frac{5}{2})\cos\left(-\frac{7\pi}{2}\right) = 0 \qquad y = r\sin\theta = (-\frac{5}{2})\sin\left(-\frac{7\pi}{2}\right) = -\frac{5}{2}$$

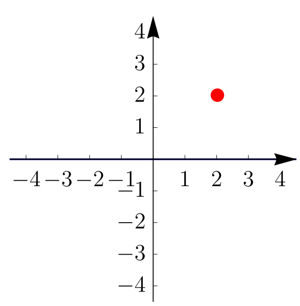
Thus, the point has rectangular form $\left(0, -\frac{5}{2}\right)$.

20.



The point $(0, -3)$ lies on the y-axis 3 units below the origin. Thus, we may take $r = 3$ and $\theta = \frac{3\pi}{2}$. Alternatively, we can take $r = -3$ and $\theta = \frac{\pi}{2}$.

22.

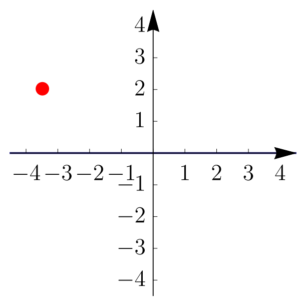


To convert $(2, 2)$ to polar coordinates, we first calculate the distance to the pole.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{2^2 + 2^2} \\ &= 2\sqrt{2} \end{aligned}$$

Since $(2, 2)$ lies in the first quadrant on the line $y = x$, it follows that $\theta = \frac{\pi}{4}$. Thus, $(2, 2)$ in rectangular coordinates is equivalent to $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ in polar coordinates. Adding π to $\frac{\pi}{4}$ and negating the radius gives us the equivalent coordinates $\left(-2\sqrt{2}, \frac{\pi}{4} + \pi\right) = \left(-2\sqrt{2}, \frac{5\pi}{4}\right)$.

24.

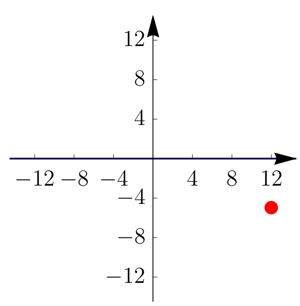


To convert $\left(-2\sqrt{3}, 2\right)$ to polar coordinates, we first calculate the distance to the pole.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-2\sqrt{3})^2 + 2^2} \\ &= \sqrt{12 + 4} \\ &= 4 \end{aligned}$$

Since $\left(-2\sqrt{3}, 2\right)$ lies in the second quadrant and $\tan\theta = \frac{2}{-2\sqrt{3}}$, it follows that $\theta = \frac{5\pi}{6}$. Thus, $\left(-2\sqrt{3}, 2\right)$ in rectangular coordinates is equivalent to $\left(4, \frac{5\pi}{6}\right)$ in polar coordinates. Adding π to $\frac{5\pi}{6}$ and negating the radius gives us the equivalent coordinates $\left(-4, \frac{11\pi}{6}\right)$.

26. The point with rectangular coordinates $(12, -5)$ is shown below.



To convert to polar form, we proceed as follows.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{12^2 + (-5)^2} \\ &= \sqrt{169} = 13 \end{aligned}$$

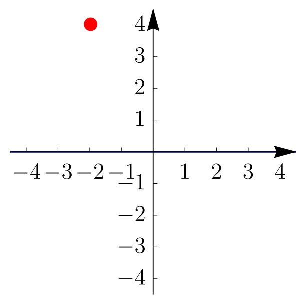
$$\tan\theta = \frac{y}{x} = \frac{-5}{12}$$

Using the $\tan^{-1}$ key on a calculator, we find that the reference angle for θ is $\tan^{-1}\left(\frac{5}{12}\right) \approx 0.3948$. Since θ is in the fourth quadrant, it follows that

$$\theta \approx 2\pi - 0.3948 \approx 5.8884$$

Thus, the approximate polar form is $(13, 5.8884)$. Subtracting π from the value for θ and negating r gives $(-13, 2.7468)$ as an equivalent form.

28. The point with rectangular coordinates $(-2, 4)$ is shown below.



To convert to polar form, we proceed as follows.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-2)^2 + 4^2} \\ &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$

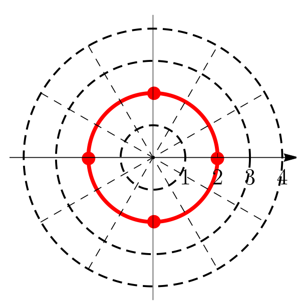
$$\tan\theta = \frac{y}{x} = \frac{4}{-2} = -2$$

Using the $\tan^{-1}$ key on a calculator, we find that the reference angle for θ is $\tan^{-1}2 \approx 1.1071$. Since θ is in the second quadrant, it follows that

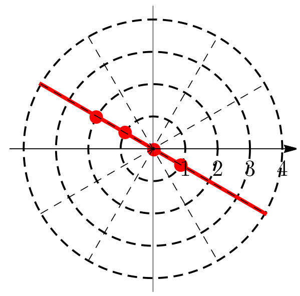
$$\theta \approx \pi - 1.1071 \approx 2.0344$$

Thus, the approximate polar form is $(2\sqrt{5}, 2.0344)$. Adding π to the value for θ and negating r gives $(-2\sqrt{5}, 5.1760)$ as an equivalent form.

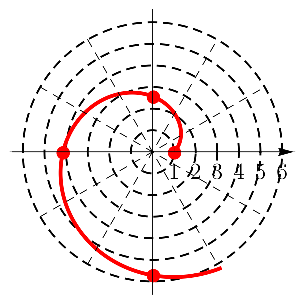
30. Each point must have $r = -2$, but θ could be any value. We choose $(-2, 0)$, $(-2, \frac{\pi}{2})$, $(-2, \pi)$, and $(-2, \frac{3\pi}{2})$. The resulting graph is a circle of radius 2, as shown below.



32. Each point must have $\theta = \frac{5\pi}{6}$, but r could be any value. We choose $(-1, \frac{5\pi}{6})$, $(0, \frac{5\pi}{6})$, $(1, \frac{5\pi}{6})$, and $(2, \frac{5\pi}{6})$. The resulting graph is a line through the pole, as shown below.

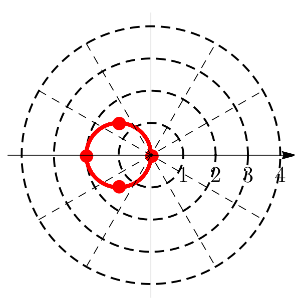


34. By chosing $\theta = 0$, $\theta = \frac{\pi}{2}$, $\theta = \pi$, and $\theta = \frac{3\pi}{2}$, we obtain the points $(1, 0)$, $(1 + \frac{\pi}{2}, \frac{\pi}{2})$, $(1 + \pi, \pi)$, and $(1 + \frac{3\pi}{2}, \frac{3\pi}{2})$. The graph of $r = 1 + \theta$ is a spiral, as shown below.



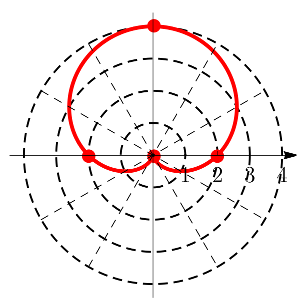
36. Choosing convenient values for θ, we produce the table shown below. Plotting the points (r, θ) and connecting with smooth curves, we obtain the circle shown.

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	-2
$\frac{\pi}{4}$	$-\sqrt{2}$
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	$\sqrt{2}$



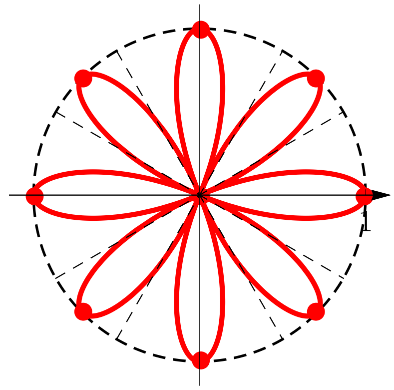
38. Choosing convenient values for θ, we produce the table shown below. Plotting the points (r, θ) and connecting with smooth curves, we obtain the cardioid shown.

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	2
$\frac{\pi}{2}$	4
π	2
$\frac{3\pi}{2}$	0



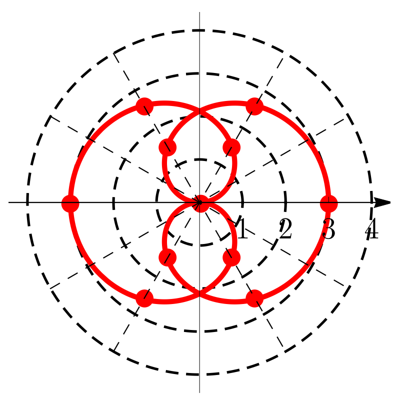
40. Choosing convenient values for θ, we produce the table shown below. Plotting the points (r, θ) and connecting with smooth curves, we obtain the graph shown.

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	1
$\frac{\pi}{8}$	0
$\frac{\pi}{4}$	-1
$\frac{3\pi}{8}$	0
$\frac{\pi}{2}$	1
$\frac{5\pi}{8}$	0
$\frac{3\pi}{4}$	-1
$\frac{7\pi}{8}$	0
π	1
$\frac{9\pi}{8}$	0
$\frac{5\pi}{4}$	-1
$\frac{11\pi}{8}$	0
$\frac{3\pi}{2}$	1
$\frac{13\pi}{8}$	0
$\frac{7\pi}{4}$	-1
$\frac{15\pi}{8}$	0
2π	1



42. Choosing convenient values for θ, we produce the table shown below. Plotting the points (r, θ) and connecting with smooth curves, we obtain the graph shown.

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	0
$\frac{\pi}{3}$	$\frac{3}{2}$
$\frac{2\pi}{3}$	$\frac{3\sqrt{3}}{2}$
π	3
$\frac{4\pi}{3}$	$\frac{3\sqrt{3}}{2}$
$\frac{5\pi}{3}$	$\frac{3}{2}$
2π	0
$\frac{7\pi}{3}$	$-\frac{3}{2}$
$\frac{8\pi}{3}$	$-\frac{3\sqrt{3}}{2}$
3π	-3
$\frac{10\pi}{3}$	$-\frac{3\sqrt{3}}{2}$
$\frac{11\pi}{3}$	$-\frac{3}{2}$
4π	0



44. Since $x = r\cos\theta$, we have

$$x = -1$$
$$r\cos\theta = -1$$

Solving for r, we have

$$\begin{aligned} r &= \frac{-1}{\cos\theta} \\ &= -\sec\theta \end{aligned}$$

46. Using the equations $x = r\cos\theta$ and $y = r\sin\theta$ and substituting gives us

$$\begin{aligned} y &= \sqrt{3}\,x \\ r\sin\theta &= \sqrt{3}\,r\cos\theta \\ \sin\theta &= \sqrt{3}\,\cos\theta \\ \tan\theta &= \sqrt{3} \\ \theta &= \tan^{-1}\sqrt{3} = \frac{\pi}{3} \end{aligned}$$

48. Using the equations $x = r\cos\theta$ and $r\sin\theta$, we have

$$\begin{aligned} x^2 - y^2 &= 1 \\ r^2\cos^2\theta - r^2\sin^2\theta &= 1 \\ r^2(\cos^2\theta - \sin^2\theta) &= 1 \\ r^2\cos 2\theta &= 1 \\ r^2 &= \frac{1}{\cos 2\theta} \\ r^2 &= \sec 2\theta \\ r &= \sqrt{\sec 2\theta} \end{aligned}$$

50. Using the equation $\tan\theta = \frac{y}{x}$, $x \neq 0$, we substitute $\theta = \frac{\pi}{4}$ to find the rectangular form.

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ \tan\frac{\pi}{4} &= \frac{y}{x} \\ 1 &= \frac{y}{x} \\ y &= x \end{aligned}$$

52. Since $x^2 + y^2 = r^2$, the equation $x^2 + y^2 = 9$ is equivalent to $r^2 = 9$, or $r = -3$.

54. Using the equations $x = r\cos\theta$ and $y = r\sin\theta$, we find that

$$\begin{aligned} r\sin\theta &= 4r\cos\theta + 2 \\ y &= 4x + 2 \end{aligned}$$

56. In order to exploit the relations $x = r\cos\theta$ and $y = r\sin\theta$, we begin by multiplying both sides of $r = 2\cos\theta + 3\sin\theta$ by r.

$$\begin{aligned} r &= 2\cos\theta + 3\sin\theta \\ (r)r &= (2\cos\theta + 3\sin\theta)r \\ r^2 &= 2r\cos\theta + 3r\sin\theta \end{aligned}$$

Substituting $x^2 + y^2$ for r^2, y for $r\sin\theta$, and x for $r\cos\theta$ gives

$$x^2 + y^2 = 2x + 3y$$

Completing the square will yield

$$\begin{aligned} x^2 - 2x + y^2 - 3y &= 0 \\ (x^2 - 2x + 1) + \left(y^2 - 3y + \frac{9}{4}\right) &= 1 + \frac{9}{4} \\ (x-1)^2 + \left(y - \frac{3}{2}\right)^2 &= \frac{13}{4} \end{aligned}$$

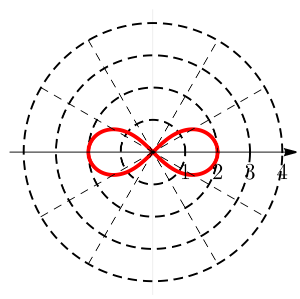
58. In order to exploit the relations $x = r\cos\theta$ and $y = r\sin\theta$, we use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ and multiply both sides of the equation by r^2.

$$\begin{aligned} r &= \sin 2\theta \\ r &= 2\sin\theta\cos\theta \\ (r^2)r &= (2\sin\theta\cos\theta)r^2 \\ r^3 &= 2(r\cos\theta)(r\sin\theta) \\ (r^2)^{3/2} &= 2(r\cos\theta)(r\sin\theta) \end{aligned}$$

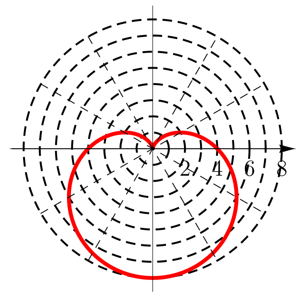
Substituting $x^2 + y^2$ for r^2, y for $r\sin\theta$, and x for $r\cos\theta$ gives

$$(x^2 + y^2)^{3/2} = 2xy$$

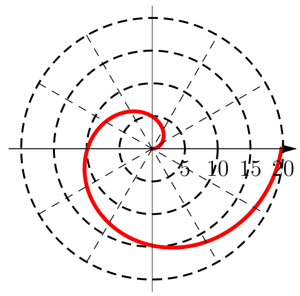
60. The graph of $r^2 = 4\cos 2\theta$ is shown below. It is a lemniscate, and so the correct answer is vi.



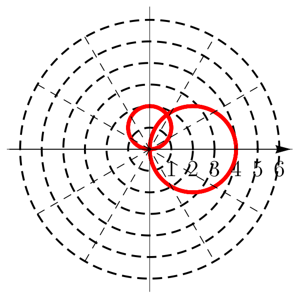
62. The graph of $r = 4(1 - \sin\theta)$ is shown below. It is a cardioid, and so the correct answer is i.



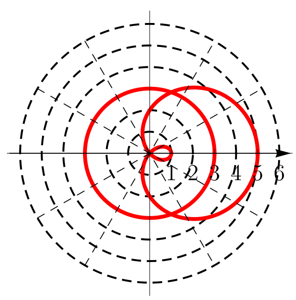
64. The graph of $r = \pi\theta$ is shown below. It is a spiral, and so the correct answer is v.



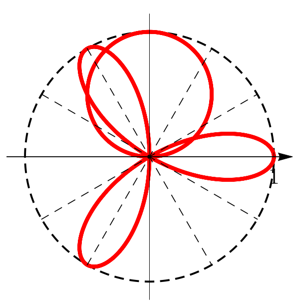
66. Using a graphing utility, we obtain the graphs of $r = 2\sin\theta$ and $r = 4\cos\theta$ shown below. Since both graphs pass through the pole, that is one point of intersection. If we restrict θ to the interval $[0, 2\pi)$, the polar coordinates of the other point of intersection is approximately $(1.79, 1.11)$.



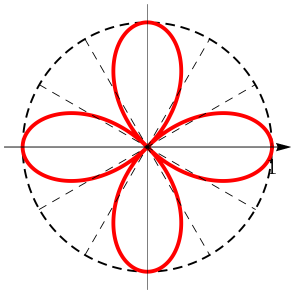
68. Using a graphing utility, we obtain the graphs of $r = 3$ and $r = 2 + 3\cos\theta$ shown below. If we restrict θ to the interval $[0, 2\pi)$, the polar coordinates of the points of intersection are approximately $(3, 1.23)$ and $(3, 5.05)$.



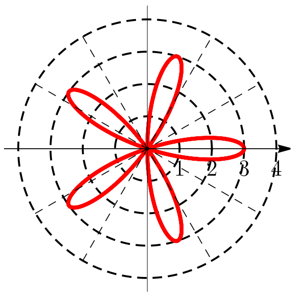
70. Using a graphing utility, we obtain the graphs of $r = \cos 3\theta$ and $r = \sin\theta$ shown below. Since both graphs pass through the pole, the pole is one point of intersection. If we restrict θ to the interval $[0, 2\pi)$, the polar coordinates of the points of the other intersection are approximately $(0.383, 0.392)$, $(0.924, 1.964)$ and $(0.707, 2.356)$.



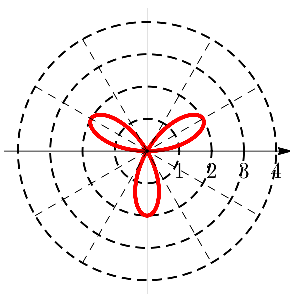
72. a. i.



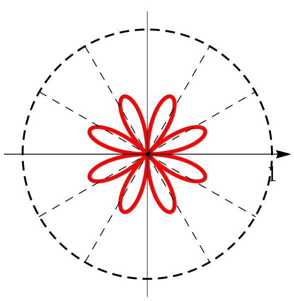
ii.



iii.



iv.



b. By looking at the examples we see that if n is odd, then the rose has n petals, while if n is even, then the rose has $2n$ petals.

c. i. $\frac{\pi}{2}$

ii. $\frac{2\pi}{5}$

iii. $\frac{2\pi}{3}$

iv. $\frac{\pi}{4}$

The smallest α for which the graph would appear the same after rotating by α is the angle between two neighboring petals. Since the petals are evenly spaced and the total angle measure of the circle is 2π, the angle $\alpha = \frac{2\pi}{p}$, where p is the number of petals. If n is odd, then $\alpha = \frac{2\pi}{n}$; if n is even, then $\alpha = \frac{\pi}{n}$.

1.5 Parametric Functions

6. Since $y = 2x$, one possible set of parametric equations is $x = t$, $y = 2t$. Once a formula is chosen for x, multiplying it by 2 will give the equation for y. Thus $x = t + 1$, $y = 2(t + 1) = 2t + 2$ is another possible set of parametric equations.

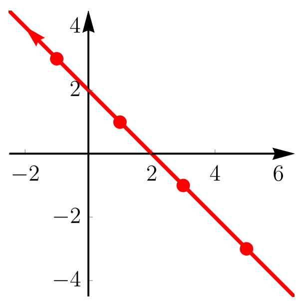
8. Since $y = f(t)$, if $x = t$ then $y = f(t)$. Thus an example of a set of parametric equations is $x = t$, $y = f(t)$.

10. To complete the table, we substitute values of t in both equations to find the corresponding values of x and y. For example, if $t = -1$ we have

$$\begin{aligned} x &= 3 - 2t \\ &= 3 - 2(-1) \\ &= 5 \end{aligned} \qquad \begin{aligned} y &= 2t - 1 \\ &= 2(-1) - 1 \\ &= -3 \end{aligned}$$

After completing the table and plotting the points, we obtain the graph shown below. The arrow indicates the direction of motion as t increases.

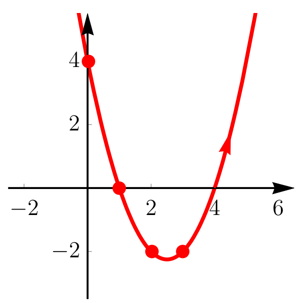
t	x	y
-1	5	-3
0	3	-1
1	1	1
2	-1	3



12. To complete the table, we substitute the given values of t into the equations $x = t + 1$ and $y = t^2 - 3t$ to find the corresponding x- and y-values.

t	x	y
-2	-1	10
-1	0	4
0	1	0
1	2	-2
2	3	-2

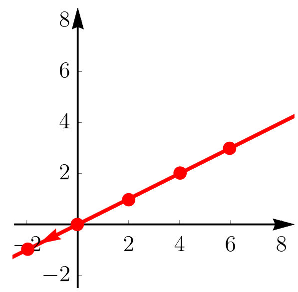
We plot the points and connect them with a smooth curve, obtaining the graph shown below. Note that the arrow indicates the direction of motion as t increases.



14. We begin by selecting some convenient t-values and computing the corresponding x- and y-values using the equations $x = 4 - 2t$ and $y = 2 - t$.

t	$x = 4 - 2t$	$y = 2 - t$
-1	6	3
0	4	2
1	2	1
2	0	0
3	-2	-1

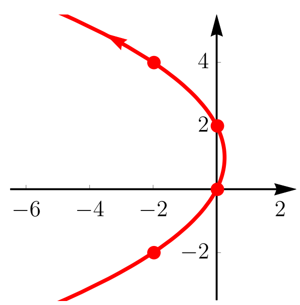
We plot the points and connect them with a smooth curve, obtaining the graph shown below. The arrow shows the direction of motion as t increases.



16. We begin by selecting some convenient t-values and computing the corresponding x- and y-values using the equations $x = -t^2 + t$ and $y = 2t$.

t	$x = -t^2 + t$	$y = 2t$
-2	-6	-4
-1	-2	-2
0	0	0
1	0	2
2	-2	4

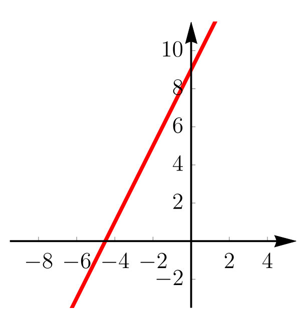
We plot the points and connect them with a smooth curve, obtaining the graph shown below. The arrow shows the direction of motion as t increases.



18. Solving $x = 3t^3 - 4$ for t^3 gives us $t^3 = \frac{x+4}{3}$. Substituting $\frac{x+4}{3}$ for t^3 in the equation for y gives

$$\begin{aligned} y &= 6t^3 + 1 \\ &= 6\left(\frac{x+4}{3}\right) + 1 \\ &= 2(x+4) + 1 \\ &= 2x + 9 \end{aligned}$$

The graph of $y = 2x + 9$ is shown below.



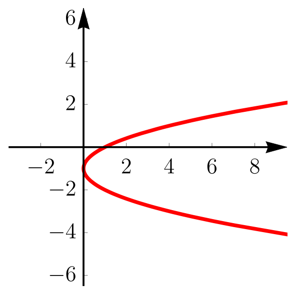
20. We have

$$\begin{aligned} x &= t^2 \\ y &= t - 1 \end{aligned}$$

Solving $y = t - 1$ for t gives $t = y + 1$. Substituting this expression for t in the equation for x gives

$$x = t^2$$
$$x = (y+1)^2$$

The graph of $x = (y+1)^2$ is shown below.



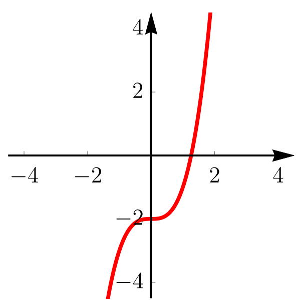
22. We have

$$x = \sqrt[3]{t}$$
$$y = t - 2$$

Solving $x = \sqrt[3]{t}$ for t gives $t = x^3$. Substituting this expression for t in the equation for y gives

$$y = t - 2$$
$$y = x^3 - 2$$

The graph of $y = x^3 - 2$ is shown below.



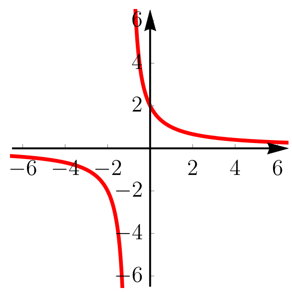
24. We have

$$x = 2t - 1$$
$$y = \frac{1}{t}$$

Solving $x = 2t - 1$ for t gives $t = \dfrac{x+1}{2}$. Substituting this expression for t in the equation for y gives

$$y = \frac{1}{t}$$
$$y = \frac{2}{x+1}$$

The graph of $y = \dfrac{2}{x+1}$ is shown below.



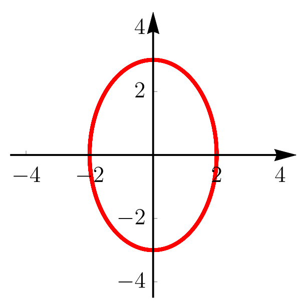
26. We have

$$x = 2\cos t$$
$$y = 3\sin t$$

Solving $x = 2\cos t$ for $\cos t$ gives $\cos t = \frac{x}{2}$. Solving $y = 3\sin t$ for $\sin t$ gives $\sin t = \frac{y}{3}$. Substituting these expressions in the identity $\sin^2 t + \cos^2 t = 1$ gives

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$$
$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

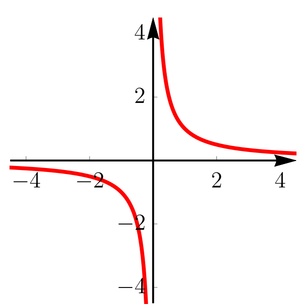
The graph of $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ is shown below.



28. We have

$$x = \tan t$$
$$y = \cot t$$

Since $\cot t = \dfrac{1}{\tan t}$, $y = \dfrac{1}{x}$ The graph of $y = \dfrac{1}{x}$ is shown below.



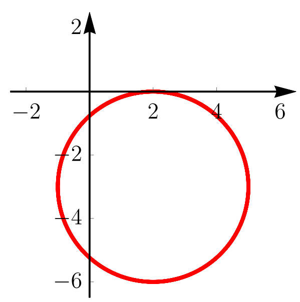
30. We have

$$x = 2 + 3\sin t$$
$$y = -3 - 3\cos t$$

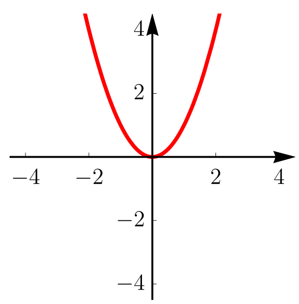
Solving $x = 2+3\sin t$ for $\sin t$ gives $\sin t = \frac{x-2}{3}$. Solving $y = -3-3\cos t$ for $\cos t$ gives $\cos t = \frac{y+3}{-3}$. Substituting these expressions in the identity $\sin^2 t + \cos^2 t = 1$ gives

$$\begin{aligned}\sin^2 t + \cos^2 t &= 1\\ \left(\frac{x-2}{3}\right)^2 + \left(\frac{y+3}{3}\right)^2 &= 1\\ \frac{(x-2)^2}{9} + \frac{(y+3)^2}{9} &= 1\\ (x-2)^2 + (y+3)^2 &= 9\end{aligned}$$

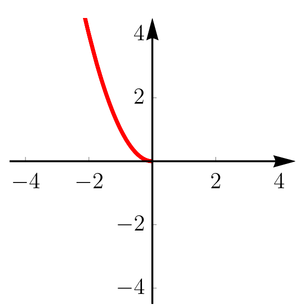
The graph of $(x-2)^2 + (y+3)^2 = 9$ is shown below.



32. a. Since $x = -t^2$, the equation $y = t^4$ is equivalent to $y = (-t^2)^2$, or $y = x^2$.

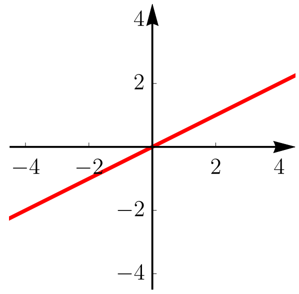


b. Using a graphing utility, we obtain the graph shown below.

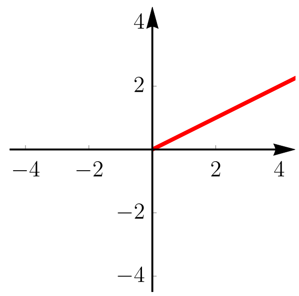


c. The difference between the graphs arises from the fact that in parametric form, $x = -t^2$ is never positive.

34. a. Since $y = |t|$, the equation $x = |2t|$ is equivalent to $x = 2|t|$, or $x = 2y$. This can be solved for y to give $y = \frac{x}{2}$. This is a line with slope $\frac{1}{2}$ and y-intercept 0, as shown.

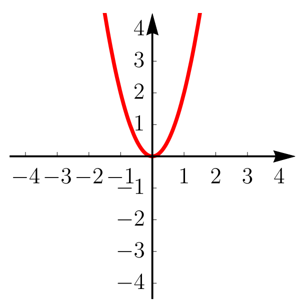


b. Using a graphing utility, we obtain the graph shown below.

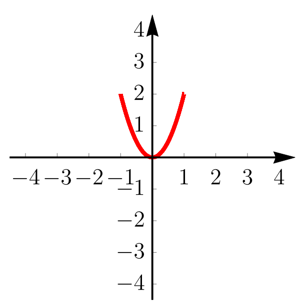


c. The difference between the graphs arises from the fact that in parametric form, $x = |2t|$ and $y = |t|$ are always nonnegative.

36. a. Since $x = \sin t$, the equation $y = 2\sin^2 t$ is equivalent to $y = 2x^2$.

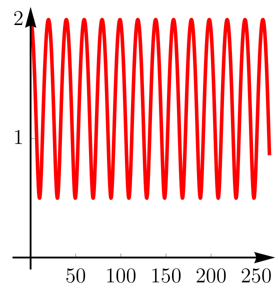


b. Using a graphing utility, we obtain the graph shown below.

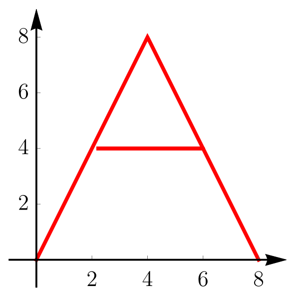


c. The difference between the graphs arises from the fact that in parametric form, $-1 \le x \le 1$ and $0 \le y \le 2$.

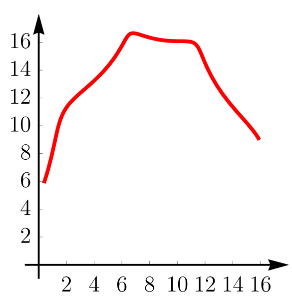
38. Using a graphing utility, we obtain the graph of $x = 0.75\sin\left(\frac{8\pi}{3}t\right) + 26.4t$, $y = 0.75\cos\left(\frac{8\pi}{3}t\right) + 1.25$, $0 \le t \le 10$ shown below. This can be interpreted as the path of a reflector strapped to a cyclist's ankle, and so the correct answer is ii.



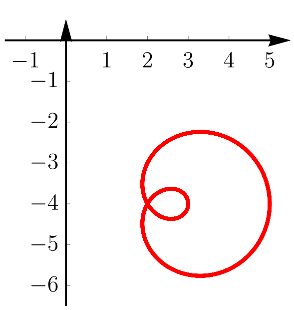
40. Using a graphing utility, we obtain the graph of $x = \frac{4}{9}|t-9| - |t-6| - |t-5| + 4|t-4| - 3|t-3| + \frac{13}{9}|t|$, $y = 2|t-6| - 2|t-5| + 2|t-3| - 4|t-2| + 2|t|$, $0 \le t \le 9$ shown below. This is clearly the letter A, and so the correct answer is iii.



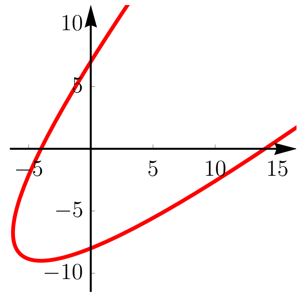
42. Using a graphing utility, we obtain the graph of $x = \frac{5}{12}\cos(4\pi t) + \frac{30\sqrt{34}}{17}t$, $y = \frac{5}{12}\sin(4\pi t) - 16t^2 + \frac{76\sqrt{34}}{17}t + 6$, $0 \le t \le 1.5$ shown below. This can be interpreted as the path of a point on a basketball, shot from the free-throw line with a slight backspin, and so the correct answer is i.



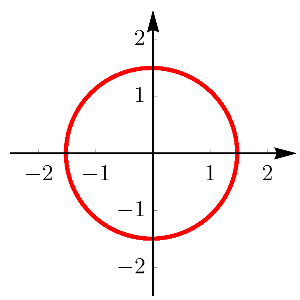
44. Using a graphing utility we can see that the crossing point is approximately $(2, -4)$.



46. Using a graphing utility, we obtain the graph shown below. The x-intercepts are approximately $(-4, 0)$ and $(14, 0)$ and the y-intercepts are approximately $(0, -8)$ and $(0, 7)$.

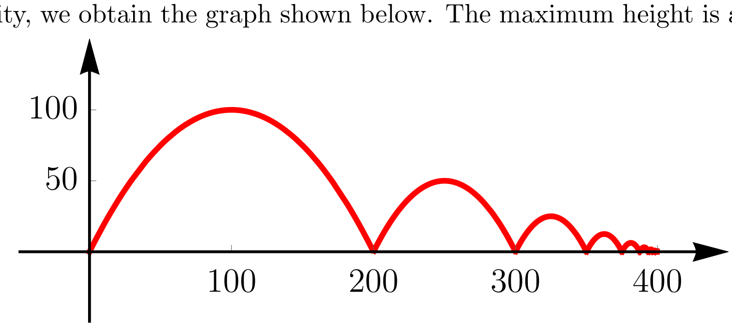


48. Using a graphing utility, we obtain the graph shown below. The perihelion and aphelion both occur when $y = 0$; that is, when $\sin(0.172t^2 - 6.459t - 3.071) = 0$. At these points $\cos(0.172t^2 - 6.459t - 3.071) = \pm 1$. The perihelion will occur when $\cos(0.172t^2 - 6.459t - 3.071) = 1$ and $x = 1.496 - 0.025 = 1.471$; the aphelion will occur when $\cos(0.172t^2 - 6.459t - 3.071) = -1$ and $x = -1.496 - 0.025 = -1.521$. Thus perihelion will occur when $0.172t^2 - 6.459t - 3.071 = 0$, and aphelion will occur when $0.172t^2 - 6.459t - 3.071 = \pi$. Solving these equations for t yields perihelion at approximately $t = 38$ and aphelion at approximately $t = 38.5$. Converting to dates gives perihelion in January 2043 and aphelion in July 2043.



50. a. The horizontal distance the ball travels can be found by noting that as t gets larger, x approaches $200(0+2) = 400$. Thus the horizontal distance the ball travels is 400 feet.

    b. Using a graphing utility, we obtain the graph shown below. The maximum height is approximately 100 feet.



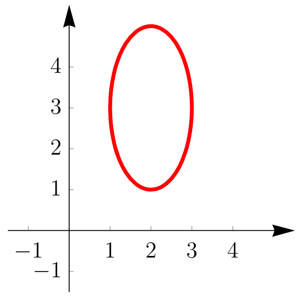
    c. Using a graphing utility, we zoom into the path of the ball after the third bounce. The maximum height is approximately 12.5 feet.

1.R Chapter 1 Review Exercises

2. This statement is true because for each given x value in the domain of the function f, there is exactly one y value.

4. This statement is true because the linear function has form $f(x) = mx + b$, which is equivalent to the slope-intercept equation of the line $y = mx + b$.

6. This statement is true. If the ellipse were centered at (h, k), and the values of a and b were both less than the lesser of $|h|$ and $|k|$, then the ellipse would have to be contained in a single quadrant. An example is shown in the graph of the ellipse $\dfrac{(x-2)^2}{1} + \dfrac{(y-3)^2}{4}$, where $h = 2$, $k = 3$, $a = 1$, and $b = 2$.

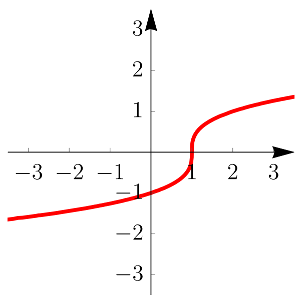


8. This statemwent is true. The two points in polar coordinates will have the same x and y coordinates in rectangular coordinates and will thus represent the same point. This follows from the trigonometric identities $\cos\theta = -\cos(\theta + \pi)$ and $\sin\theta = -\sin(\theta + \pi)$:

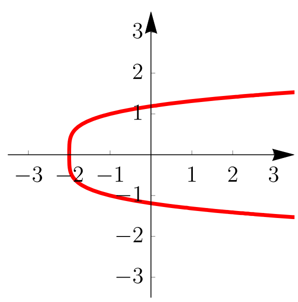
$$x = r\cos\theta = r(-\cos(\theta + \pi)) = (-r)\cos(\theta + \pi)$$
$$y = r\sin\theta = r(-\sin(\theta + \pi)) = (-r)\sin(\theta + \pi)$$

10. This statement is true. Let c be the x-coordinate of the x-intercept that f and g have in common. Then $f(c) = g(c) = 0$. If we let $t = c$ in the parametric curve $x = f(t), y = g(t)$, that at $t = c$ the curve we be at the point $(f(c), g(c)) = (0, 0)$.

12. The equation $x - y^3 = 1$ defines y as a function of x since its graph satisfies the vertical line test:



14. The equation $x = y^4 - 2$ does not define y as a function of x since its graph does not satisfy the vertical line test:



16. a. Substituting 0 in for x in the formula for $h(x)$ gives

$$h(0) = \frac{0}{2(0) + 5} = \frac{0}{5} = 0$$

b. Substituting -5 in for x in the formula for $h(x)$ gives

$$\begin{aligned} h(-5) &= \frac{-5}{2(-5)+5} \\ &= \frac{-5}{(-10)+5} \\ &= \frac{-5}{-5} \\ &= 1 \end{aligned}$$

18. a. Substituting $x-2$ for x in the formula for $g(x)$, we have

$$\begin{aligned} g(x-2) &= [(x-2)+2]^2 \\ &= (x-2+2)^2 \\ &= x^2 \end{aligned}$$

b. Substituting $\frac{1}{x}$ for x in the formula for $g(x)$, we have

$$\begin{aligned} g\left(\frac{1}{x}\right) &= \left(\frac{1}{x}+2\right)^2 \\ &= \left(\frac{1}{x}+\frac{2x}{x}\right)^2 \\ &= \left(\frac{1+2x}{x}\right)^2 \\ &= \left(\frac{2x+1}{x}\right)^2 \end{aligned}$$

20. a. Substituting -2 for x in the formula for $f(x)$, we have

$$\begin{aligned} f(-2) &= \frac{|-2-3|}{-2-3} \\ &= \frac{|-5|}{-5} \\ &= \frac{5}{-5} \\ &= -1 \end{aligned}$$

b. Substituting 10 for x in the formula for $f(x)$, we have

$$\begin{aligned} f(10) &= \frac{|10-3|}{10-3} \\ &= \frac{|7|}{7} \\ &= \frac{7}{7} \\ &= 1 \end{aligned}$$

c. Substituting c for x in the formula for $f(x)$ and noting that $c-3<0$ since $c<3$, we have

$$\begin{aligned} f(c) &= \frac{|c-3|}{c-3} \\ &= \frac{-(c-3)}{c-3} \\ &= -1 \end{aligned}$$

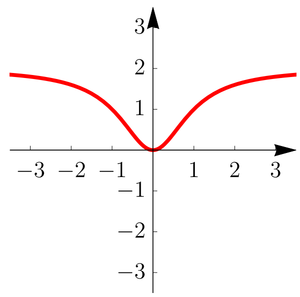
22. Since we are only considering values of x for which $g(x)$ is real, the valid input values of x are those for which $3 - x \geq 0$. Solving for x gives $x \leq 3$. Therefore, the domain of g is $(-\infty, 3]$.

24. Since we are only considering the input values for which the function is defined, the domain includes all values of x except those for which the denominator is 0. To determine the values that must be excluded, we set $4 - x^2 = 0$ and solve for x, as follows.

$$4 - x^2 = 0$$
$$(2 - x)(2 + x) = 0$$
$$x = 2,\ x = -2$$

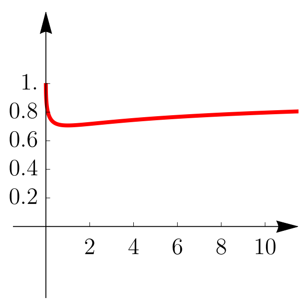
Thus, the domain of f is $\{x | x \neq 2, x \neq -2\}$.

26.



The domain is the set of x-coordinates of points on the graph. From the graph shown above, we see that the domain appears to be $(-\infty, \infty)$. The range is the set of y-coordinates of points on the graph. Again, using the graph above it appears that the range is $[0, 2)$.

28.



The domain is the set of x-coordinates of points on the graph. From the graph shown above, we see that the domain appears to be $[0, \infty)$. The endpoint can be confirmed since $g(0) = 1$. The range is the set of y-coordinates of points on the graph. Again, using the graph above it appears that the range is approximately $(0.7, 1]$. Again the endpoint can be confirmed since $g(0) = 1$.

30. The first five terms of the sequence $a_n = n + \frac{(-1)^n}{n}$ are:

$$a_1 = 1 + \frac{(-1)^1}{1} = 1 - 1 = 0$$
$$a_2 = 2 + \frac{(-1)^2}{2} = 2 + \frac{1}{2} = \frac{5}{2}$$
$$a_3 = 3 + \frac{(-1)^3}{3} = 3 - \frac{1}{3} = \frac{8}{3}$$
$$a_4 = 4 + \frac{(-1)^4}{4} = 4 + \frac{1}{4} = \frac{17}{4}$$
$$a_5 = 5 + \frac{(-1)^5}{5} = 5 - \frac{1}{5} = \frac{24}{5}$$

32. The first five terms of the sequence $b_n = \dfrac{n+3}{2}$ are:
$b_1 = \dfrac{1+3}{2} = \dfrac{4}{2} = 2$
$b_2 = \dfrac{2+3}{2} = \dfrac{5}{2}$
$b_3 = \dfrac{3+3}{2} = \dfrac{6}{2} = 3$
$b_4 = \dfrac{4+3}{2} = \dfrac{7}{2}$
$b_5 = \dfrac{5+3}{2} = \dfrac{8}{2} = 4$

34. The first five terms of the sequence $a_n = \frac{2^n}{3^n}$ are:
$a_1 = \dfrac{2^1}{3^1} = \dfrac{2}{3}$
$a_2 = \dfrac{2^2}{3^2} = \dfrac{4}{9}$
$a_3 = \dfrac{2^3}{3^3} = \dfrac{8}{27}$
$a_4 = \dfrac{2^4}{3^4} = \dfrac{16}{81}$
$a_5 = \dfrac{2^5}{3^5} = \dfrac{32}{243}$

36. The first five terms of the sequence are:
$u_1 = 256$
$u_2 = \sqrt{u_1} = \sqrt{256} = 16$
$u_3 = \sqrt{u_2} = \sqrt{16} = 4$
$u_4 = \sqrt{u_3} = \sqrt{4} = 2$
$u_5 = \sqrt{u_4} = \sqrt{2}$

38. We want to define a_n so that

$$a_1 = e,\ a_2 = 2e^2,\ a_3 = 3e^3,\ a_4 = 4e^4$$

Since each term is e times the previous term with a coefficient equal to the number of the term, and the first term is e, we have $a_n = ne^n$.

40. We want to define a_n so that

$$a_1 = -3,\ a_2 = 2,\ a_3 = 7,\ a_4 = 12$$

Notice that each term is five more than the preceding term, which implies that the term should be something like $b_n = 5n$. The sequence b_n begins $5, 10, 15, 20$ and we can see that the sequence we desire is eight less than each term in the sequence b_n, so we have that $a_n = b_n - 8 = 5n - 8$.

42. Since each successive term in the sequence is decreasing by the same factor, this is a geometric sequence. The common ratio is $r = -\dfrac{1}{4}$, which means that to find each new term, we multiply the previous term by $-\dfrac{1}{4}$. Therefore, a formula for the nth term is $a_n = 24\left(-\dfrac{1}{4}\right)^{n-1} = \dfrac{24}{(-4)^{n-1}}$.

44. The numerator of each term is the number of the term, and the denominator of each term is one greater than the numerator, which suggests a form of $\dfrac{n}{n+1}$. However, the signs of the terms also alternate beginning with a negative sign, so the complete expression for the nth term is $\frac{(-1)^{n+1}n}{n+1}$.

46. As the function has the form $g(x) = mx + b$, it is a linear function.

48. This function is a polynomial function with degree greater than 2, so it neither linear, nor quadratic, nor piecewise-defined.

50. Piecewise-defined

    As this function uses different rules for different portions of its domain, it is a piecewise-defined function.

52. Since g is linear, $g(x) = mx + b$ for some m and b. Since the y-intercept is -2, $b = -2$ and $g(x) = mx - 2$. Plugging the values $x = 1$ and $g(1) = 5$ into this equation gives $5 = m(1) - 2$, or $m = 7$. Thus $g(x) = 7x - 2$.

54. Since f is a quadratic function with a vertical axis of symmetry, it is of the form $f(x) = a(x - h)^2 + k$, where the point (h, k) is the vertex of the parabola whose graph is $y = f(x)$. Since $x = -1$ is the axis of symmetry, the x-coordinate of the vertex is $h = -1$. Since $f(-1) = 4$, the y-coordinate of the vertex is $k = 4$. Thus $f(x) = a(x + 1)^2 + 4$. Since $f(0) = 3$, we can plug these values into the equation, giving $3 = a(1)^2 + 4$, or $a = -1$. Thus $f(x) = -(x + 1)^2 + 4 = -x^2 - 2x + 3$.

56. Since the graph passes through $(2, 3)$, we know that $3 = \sqrt{2a + b}$, or $2a + b = 9$. Solving for b we find that $b = 9 - 2a$. Since the graph passes through $(10, 5)$, we know that $5 = \sqrt{10a + b}$, or $10a + b = 25$. Since $b = 9 - 2a$, we find that

$$25 = 10a + b = 10a + 9 - 2a = 8a + 9$$

    Thus $8a = 16$, $a = 2$, and $b = 9 - 2(2) = 5$. The equation is therefore $h(x) = \sqrt{2x + 5}$.

58. Using a graphing utility, a CAS, or a spreadsheet, we obtain the following: $y = 2.39 - 0.85x$.

60. Using a graphing utility, a CAS, or a spreadsheet, we obtain the following: $y = 204.54 - 9.38x$. Thus when $x = 9$, $y \approx 204.54 - 9.38(9) = 120.58$.

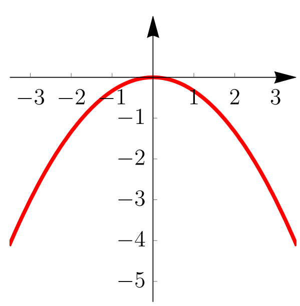
62. The standard form for the equation of a parabola with vertex (h, k) and a horizontal axis of symmetry is $x - h = a(y - k)^2$. We are given the equation

$$x^2 + 3y = 0$$

    which may be rewtirren in the form

$$y - 0 = -\frac{1}{3}(x - 0)^2$$

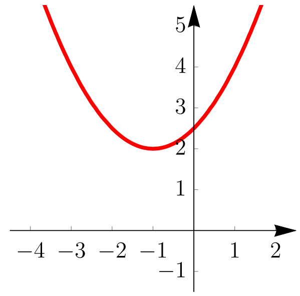
    In this case, both h and k are 0, and so the vertex is the point $(0, 0)$ and the axis of symmetry is the horizontal line $y = 0$. The graph is shown below.



64. The standard equation of a parabola with a vertical axis of symmetry is $y - k = a(x - h)^2$. We have

$$y - 2 = \frac{1}{2}(x + 1)^2$$

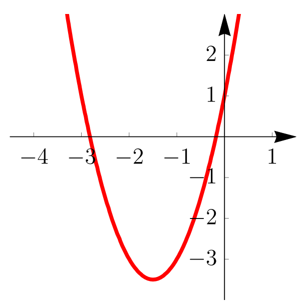
    Thus, the vertex is the point $(-1, 2)$, and the axis of symmetry is the vertical line $x = -1$. The graph of the parabola is shown below.



66. By rewriting the equation of the parabola in standard form, we can find the coordinates of the vertex.

$$\begin{aligned} y &= 2x^2 + 6x + 1 \\ y - 1 &= 2x^2 + 6x \\ y - 1 &= 2(x^2 + 3x + \boxed{}) \\ y - 1 + \frac{9}{2} &= 2(x^2 + 3x + \boxed{\tfrac{9}{4}}) \\ y + \frac{7}{2} &= 2\left(x + \frac{3}{2}\right)^2 \end{aligned}$$

From the standard form, we can see that the vertex of the parabola is $\left(-\frac{3}{2}, -\frac{7}{2}\right)$, and its axis of symmetry is vertical. Using a graphing utility (or plotting a few points and connecting with a smooth curve), we obtain the graph shown below.



68. The standard equation of a parabola with a horizontal axis of symmetry is $x - h = a(y - k)^2$. Since the vertex is $(4, 3)$, we have $h = 4$ and $k = 3$, so

$$x - 4 = a(y - 3)^2$$

Using the fact that $(2, 4)$ is on the parabola gives us

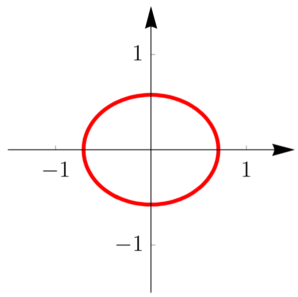
$$\begin{aligned} 2 - 4 &= a(4 - 3)^2 - 2 & &= a1^2 \\ -2 &= a \end{aligned}$$

So the equation of the parabola is $x - 4 = -2(y - 3)^2$.

70. First, we write the equation in standard form:

$$\begin{aligned} 2x^2 + 3y^2 &= 1 \\ \frac{x^2}{1/2} + \frac{y^2}{1/3} &= 1 \end{aligned}$$

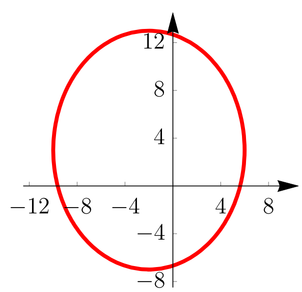
Thus, $a = \frac{1}{\sqrt{2}}$ and $b = \frac{1}{\sqrt{3}}$. Since $a > b$, the graph has a horizontal orientation. The center of the ellipse is at $(0,0)$, and so the major axis is horizontal with vertices $\left(\frac{1}{\sqrt{2}}, 0\right)$, $\left(-\frac{1}{\sqrt{2}}, 0\right)$. The minor axis connects $\left(0, \frac{1}{\sqrt{3}}\right)$, $\left(0, -\frac{1}{\sqrt{3}}\right)$. The graph is shown below.



72. We have

$$\begin{aligned} \frac{(x+2)^2}{64} + \frac{(y-3)^2}{100} &= 1 \\ \frac{(x+2)^2}{8^2} + \frac{(y-3)^2}{10^2} &= 1 \end{aligned}$$

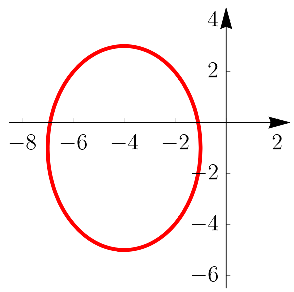
The center of the ellipse is $(-2,3)$. We see that $a = 8$, $b = 10$, and $a < b$, and so the major axis is vertical and connects the vertices $(-2,-7)$ and $(-2,13)$. The minor axis connects $(-10,3)$ and $(6,3)$. The graph is shown below.



74. By finding the standard equation of an ellipse from the given information, we can identify its center, vertices, and major and minor axes.

$$\begin{aligned} 16x^2 + 9y^2 + 128x + 18y + 121 &= 0 \\ (16x^2 + 128x) + (9y^2 + 18y) &= -121 \\ 16(x^2 + 8x) + 9(y^2 + 2y) &= -121 \\ 16(x^2 + 8x + 16) + 9(y^2 + 2y + 1) &= -121 + 16(16) + 9(1) \\ 16(x+4)^2 + 9(y+1)^2 &= 144 \\ \frac{(x+4)^2}{9} + \frac{(y+1)^2}{16} &= 1 \\ \frac{(x+4)^2}{3^2} + \frac{(y+1)^2}{4^2} &= 1 \end{aligned}$$

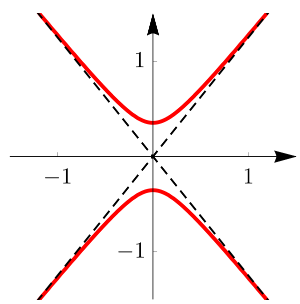
From the standard equation, we see that the center is $(-4,-1)$, and that $a = 3$ and $b = 4$. Since $a < b$, the ellipse has a vertical major axis with vertices $(-4,-5)$ and $(-4,3)$. The minor axis extends from $(-7,-1)$ to $(-1,-1)$. The graph is shown below.



76. We begin by rewriting the equation in standard form.

$$8y^2 - 12x^2 = 1$$
$$\frac{y^2}{1/8} - \frac{x^2}{1/12} = 1$$
$$\frac{y^2}{(1/\sqrt{8})^2} - \frac{x^2}{(1/\sqrt{12})^2} = 1$$

The center of the hyperbola is $(0,0)$. The transverse axis is vertical with $a = \frac{1}{\sqrt{12}}$ and $b = \frac{1}{\sqrt{8}}$. The vertices are located $\frac{1}{\sqrt{8}}$ units above and below of the center, at $(0 - \frac{1}{\sqrt{8}})$ and $(0, \frac{1}{\sqrt{8}})$. The asymptotes for a hyperbola centered at the origin are $y = \pm\frac{b}{a}x$. Therefore, for this hyperbola the asymptotes are $y = \pm\sqrt{\frac{3}{2}}x$. The graph is shown below.



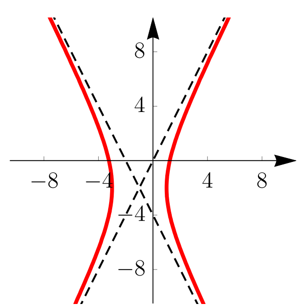
78. The standard equation of a hyperbola with a horizontal transverse axis centered at (h,k) is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Thus, the graph of $\frac{(x+1)^2}{4} - \frac{(y+2)^2}{16} = 1$ is centered at $(-1,-2)$ with $a = 2$ and $b = 4$. It follows that the vertices are 2 units to the left and right of the center at $(-3,-2)$ and $(1,-2)$. The asymptotes of a hyperbola with a horizontal transverse axis centered at (h,k) are

$$y = \pm\frac{b}{a}(x-h) + k$$

so the asymptotes for this hyperbola are $y = \pm 2(x+1) - 2$. The graph is shown below.



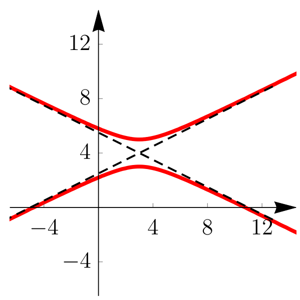
80. We begin by rewriting the equation in standard form by completing the square on the x and y terms.

$$x^2 - 4y^2 - 6x + 32y = 51$$
$$x^2 - 6x - 4y^2 + 32y = 51$$
$$(x^2 - 6x) - 4(y^2 - 8y) = 51$$
$$(x^2 - 6x + 9) - 4(y^2 - 8y + 16) = -4$$
$$\frac{(y-4)^2}{1} - \frac{(x-3)^2}{4} = 1$$

The hyperbola is centered at $(3, 4)$. The axis is vertical with $a = 2$ and $b = 1$. The vertices are 1 unit above and below the center, at $(3, 3)$ and $(3, 5)$. The asymptotes of a hyperbola with a vertical transverse axis centered at (h, k) are

$$y = \pm\frac{b}{a}(x - h) + k$$

so the asymptotes for this hyperbola are $y = \pm\frac{1}{2}(x - 3) + 4$. The graph is shown below.



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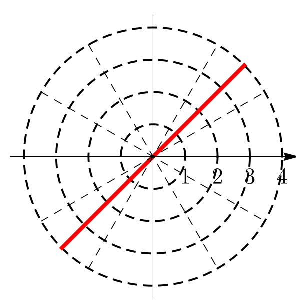

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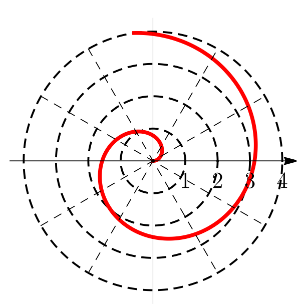


86. Using the equations $x = r\cos\theta$ and $y = r\sin\theta$ and substituting gives us

$$\begin{aligned} y &= 3\,x \\ r\sin\theta &= 3\,r\cos\theta \\ \sin\theta &= 3\,\cos\theta \\ \tan\theta &= 3 \\ \theta &= \tan^{-1} 3 \end{aligned}$$

88. Using the equations $x = r\cos\theta$ and $r\sin\theta$, we have

$$\begin{aligned} x^2 + (y-2)^2 &= 4 \\ r^2\cos^2\theta + (r\sin\theta - 2)^2 &= 4 \\ r^2\cos^2\theta + r^2\sin^2\theta - 4r\sin\theta + 4 &= 4 \\ r^2(\cos^2\theta + \sin^2\theta) - 4r\sin\theta &= 0 \\ r^2 &= 4r\sin\theta \\ r &= 4\sin\theta \end{aligned}$$

90. Since $x^2 + y^2 = r^2$, the equation $x^2 + y^2 = 25$ is equivalent to $r^2 = 25$, or $r = 5$.

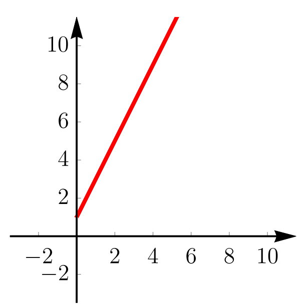
92. Using the fact that $\csc\theta = \frac{1}{\sin\theta}$ and $y = r\sin\theta$, we find that

$$\begin{aligned} r &= 2\csc\theta \\ r &= \frac{2}{\sin\theta} \\ r\sin\theta &= 2 \\ y &= 2 \end{aligned}$$

94. Solving $y = 2t^2 + 1$ for t^2 gives us $t^2 = \frac{y-1}{2}$. Substituting $\frac{y-1}{2}$ for t^2 in the equation for x gives

$$\begin{aligned} x &= \frac{y-1}{2} \\ 2x &= y - 1 \\ y &= 2x + 1 \end{aligned}$$

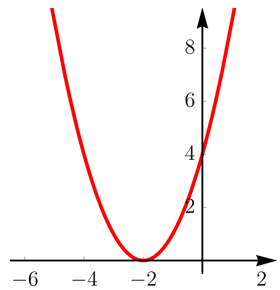
The graph of $y = 2x + 1$ is shown below. Notice that since $x = t^2 \geq 0$, only that part of the line with $x \geq 0$ is shown.



96. Solving $x = t - 2$ for t gives us $t = x + 2$. Substituting $x + 2$ for t in the equation for y gives

$$\begin{aligned} y &= t^2 \\ y &= (x+2)^2 \end{aligned}$$

The graph of $y = (x+2)^2$ is shown below.



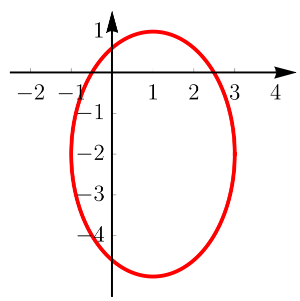
98. We have

$$x = 1 + 2\cos t$$
$$y = -2 + 3\sin t$$

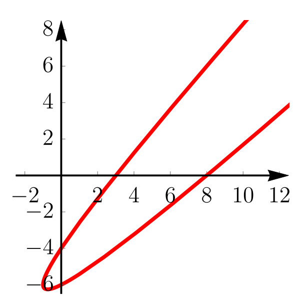
Solving $x = 1+2\cos t$ for $\cos t$ gives $\cos t = \frac{x-1}{2}$. Solving $y = -2+3\sin t$ for $\sin t$ gives $\sin t = \frac{y+2}{3}$. Substituting these expressions in the identity $\sin^2 t + \cos^2 t = 1$ gives

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{y+2}{3}\right)^2 + \left(\frac{x-1}{2}\right)^2 = 1$$
$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

The graph of $\dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{9} = 1$ is shown below.

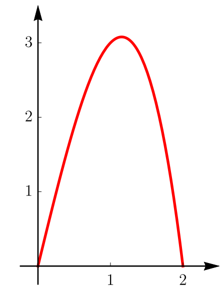


100. The graph is



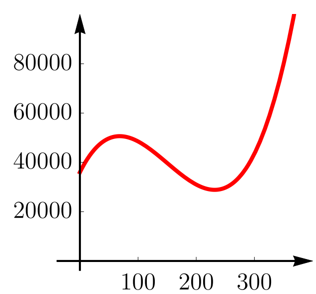
We see that approximate value for the x-intercepts are 3 and 8, while approximate values for the y-intercepts are -6 and -4.

102. The area a rectangle is its length multiplied by its height. The length of the shaded rectangle is x while its height is $4 - x^2$, so its area is $f(x) = x(4 - x^2) = 4x - x^3$. A graph of this function is shown here.

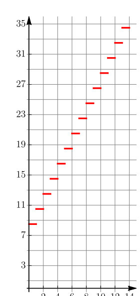


From this graph it seems that a value of approximately $x = 1.1$ will yield the largest possible area.

104. As we move from left to right on the graph shown below, the graph is rising in the interval $(0, 70)$, so C is increasing on that interval. The graph is falling as we move from left to right in the interval $(70, 230)$, so C is decreasing on that interval. The graph is rising as we move from left to right in the interval $(230, \infty)$, so C is increasing on that interval. A minimum of C occurs at approximately $x = 230$, so that is the number of units that must be manufactured to minimize the cost.



106. The graph of C is shown below. Examining the graph yields the observation that the heaviest package that can be sent for a cost not excceding \$31 is 12 pounds.



108. The least-squares best fit line is $18.02 - 0.52x$. The correlation coefficient for this model is $R^2 = 0.97$. The model estimates death rates of -0.13 in 2005 and -2.72 in 2010. These estimates are negative, which is not a reasonable answer.

110. The least-squares best exponential fit is $4.0e^{0.03x}$. The correlation coefficient for this model is $R^2 > 0.999$. In 1850, $x = 60$ and this model estimates a population of 24.2 million. This estimate is one million people (or about 4.3%) more than the actual population.

112. a. Since the path of the ball is a parabola, we can find the maximum height by finding the y-coordinate of its

vertex. We complete the square to find:

$$\begin{aligned}
450y &= -x^2 + 450x + 1350 \\
y &= -\frac{1}{450}x^2 + x + 3 \\
y - 3 &= -\frac{1}{450}(x^2 - 450x) \\
y - 3 - \frac{50625}{450} &= -\frac{1}{450}(x^2 - 450x + 50625) \\
y - \frac{231}{2} &= -\frac{1}{450}(x - 225)^2
\end{aligned}$$

Thus the y-coordinate of the vertex is $\frac{231}{2} = 115.5$, so the maximum height of the ball is 115.5 feet.

b. The ball will land when its y coordinate is zero. We solve $y = -\frac{1}{450}x^2 + x + 3 = 0$ using the quadratic formula to find that $x \approx 453$, so the ball lands approximately 453 feet from home plate.