(c) 2 y
0
2 4 x
_2
(d)
0
2 4 x
4
_4
_8
y (e)
0
2 4 x
_4
_8
y

5. (a) . *f* ± *g*/ .2/ Δ *f* .0/ Δ 1
(b) .*g* ± *f* / .2/ Δ *g* .3/ Δ 1
(c) . *f* ± *f* / .2/ Δ *f* .3/ Δ 4
(d) .*g* ± *g*/ .2/ Δ *g* .0/ Δ 4
(e) . *f* X *g*/ .2/ Δ *f* .2/ X *g* .2/ Δ 3 X 0 Δ 3
(f) ∝ *gf* ∂ .2/ is undefined because *g* .2/ Δ 0.
(g) *g*Υ1 .2/ is undefined because *g* .*x*/ takes on the value 2 twice, for *x* Δ 0:6 and *x* Δ 3:4.

6. *f* .*x*/ Δ
8<:
Υ*x* Υ 1 if *x* < Υ1
π1 Υ x^2 if Υ1 • *x* • 1
Υ*x* X 1 if *x* > 1

7. (a) *g* .*x*/ Δ $3x^2$ X 3*x*
(b) *g* .4/ Δ 60, *f* .4/ Δ 59:023837
(c) The percentage error in using *g* .4/ as an approximation for *f* .4/ is 100
↓↓↓↓
f .4/ Υ *g* .4/
f .4/
↓↓↓↓
Δ 1:65%.
(d) For larger values of *x*, *g* .*x*/ is an overestimate of *f* .*x*/ because the coefficient of the dominant term (x^2) is larger.

8. *f* Υ1 .*x*/ Δ x^5 X $2x^3$ X 3*x* X 1
(a) *f* Υ1 .1/ Δ 7, *f* .1/ Δ 0
(b) The value x_0 such that *f* .x_0/ Δ 1 is *f* Υ1 .1/ Δ 7.
(c) The value y_0 such that *f* Υ1 .y_0/ Δ 1 is *f* .1/ Δ 0.

(d) The graph of *f* .*x*/ is the graph of *f* Υ1 .*x*/ reflected about the line *y* Δ *x*.
x
0
f
y
fÐ!

9. Let *f* .*x*/ Δ 3 .2/Υ*x* X 1: Then *f* .*x*/ is always decreasing, has a horizontal asymptote at *y* Δ 1, and *f* .0/ Δ 4:

10. *f* .*x*/ Δ 1:2 .2/0:585*x*; *A* Δ 1:2, *k* Δ 0:585
11. (a) *f* .50; 000/ ϒ *f* .49; 999/ represents the cost of producing the 50,000th disc.
(b) *f* ϒ1 .10/ represents the number of discs that can be made for $10,000:
(c) The cost per disc is cheapest for 30; 000 < *a* < 40; 000: This is where the slope of *f* is the smallest.
(d) One possible explanation for the sudden increase in the curve's slope is scarcity of materials.

2 Limits and Derivatives

2.1 TheTangent and Velocity Problems

SUGGESTED TIME AND EMPHASIS

12
–1 class Essential material

POINTS TO STRESS

1. The tangent line viewed as the limit of secant lines.
2. The concepts of average versus instantaneous velocity, described numerically, visually, and in physical terms.
3. The tangent line as the line obtained by "zooming in" on a smooth function; local linearity.
4. Approximating the slope of the tangent line using slopes of secant lines.

QUIZ QUESTIONS

″ **TEXT QUESTION** Geometrically, what is "the line tangent to a curve" at a particular point?
ANSWER There are different correct ones. Examples include the best linear approximation to a curve at a point, or the result of repeated "zooming in" on a curve.

″ **DRILL QUESTION** Draw the line tangent to the following curve at each of the indicated points:

y=f(x)
x

y

ANSWER

y=f(x)
x

y

MATERIALS FOR LECTURE

″ Point out that if a car is driving along a curve, the headlights will point along the direction of the tangent line.

″ Discuss the phrase "instantaneous velocity." Ask the class for a definition, such as, "It is the limit of average velocities." Use this discussion to shape a more precise definition of a limit.

″ Illustrate that many functions such as x^2 and *x* ϒ 2 sin *x* look locally linear, and discuss the relationship of this property to the concept of the tangent line. Then pose the question, "What does a secant line to a linear function look like?"